Chapter.3
Static Electric Fields

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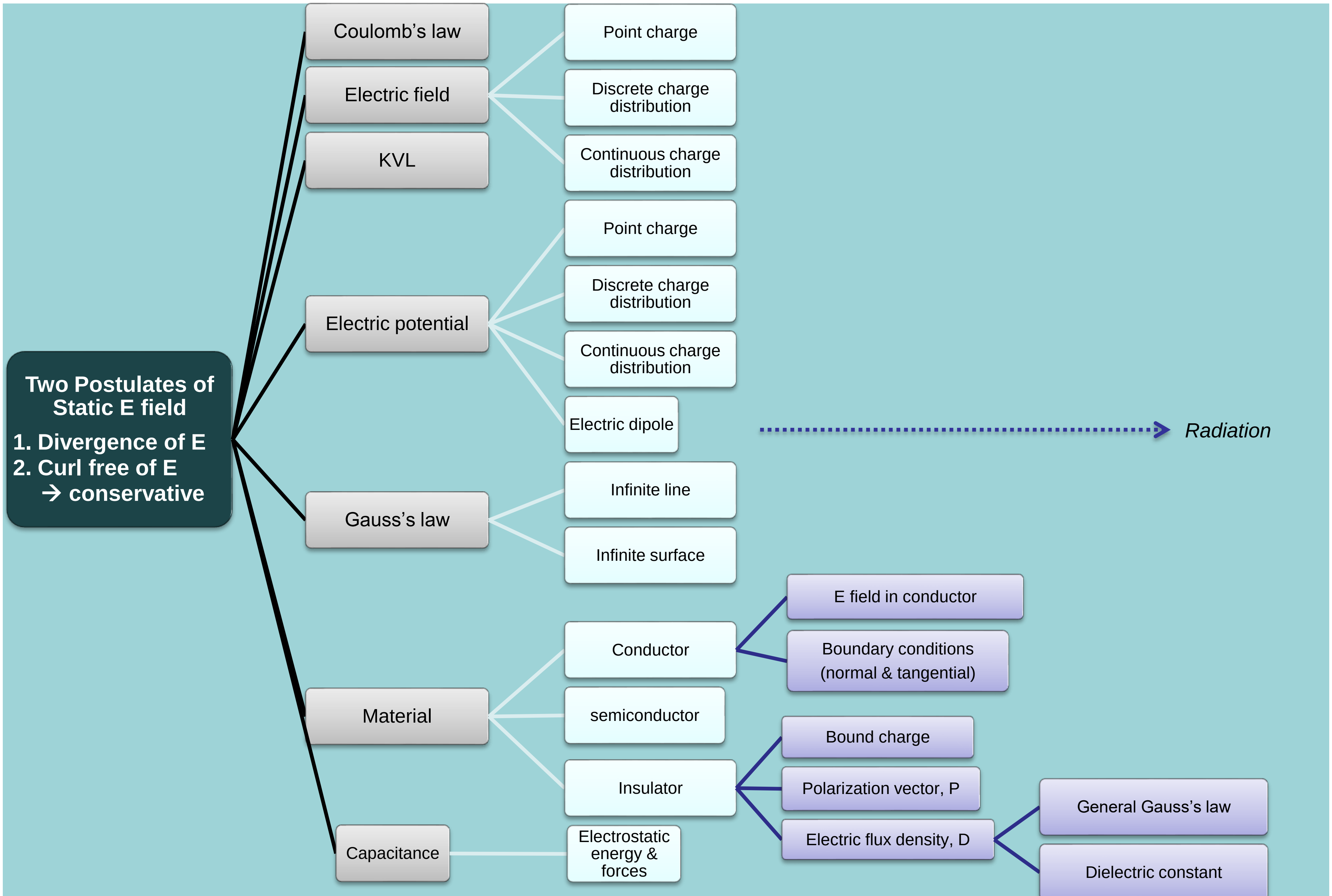
Overview

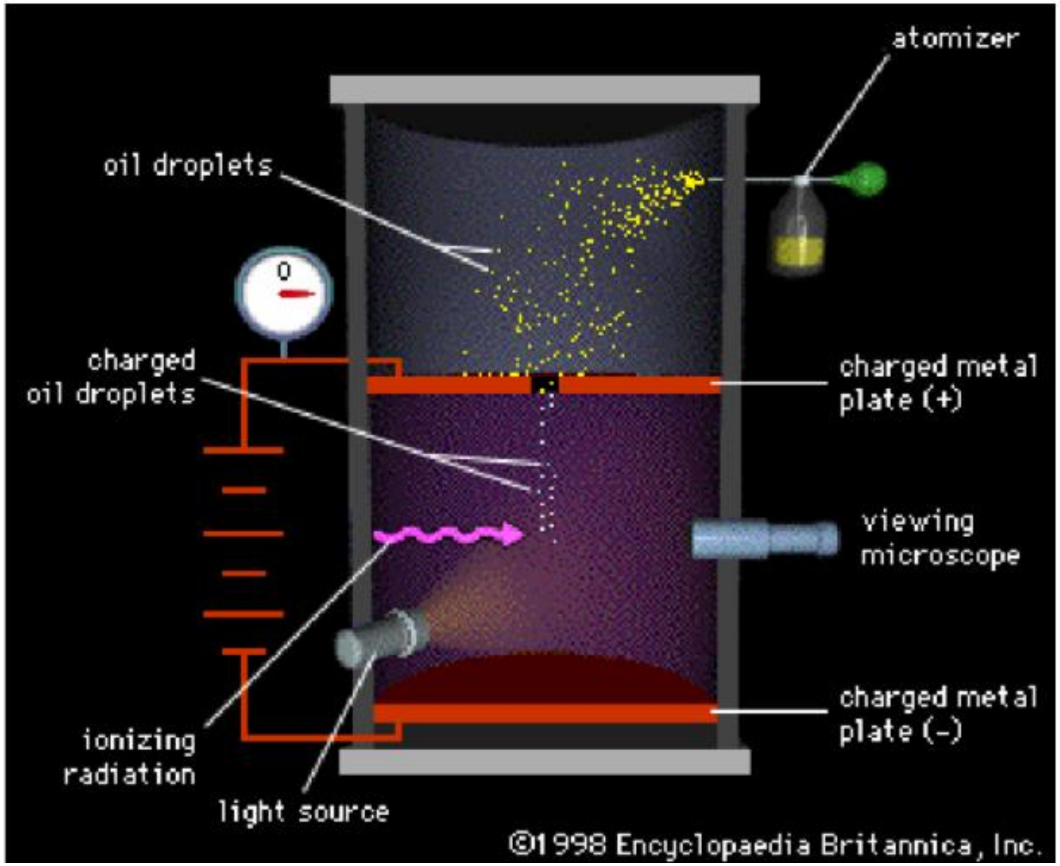


Two Postulates of Static E field
1. Divergence of E
2. Curl free of E
→ conservative
Coulomb's law
Electric field
KVL
Electric potential
Gauss's law
Material
Capacitance
Point charge
Discrete charge distribution
Continuous charge distribution
Point charge
Discrete charge distribution
Continuous charge distribution
Electric dipole
Radiation
Infinite line
Infinite surface
Conductor
semiconductor
Insulator
Electrostatic energy & forces
E field in conductor
Boundary conditions (normal & tangential)
Bound charge
Polarization vector, P
Electric flux density, D
General Gauss's law
Dielectric constant

Millikan's Exp - electron charge

$$e/m = \text{ J. J. Thomson}$$

$$e = ? \rightarrow m = ?$$

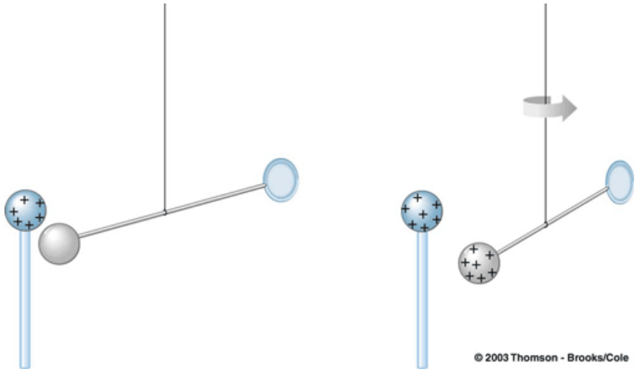


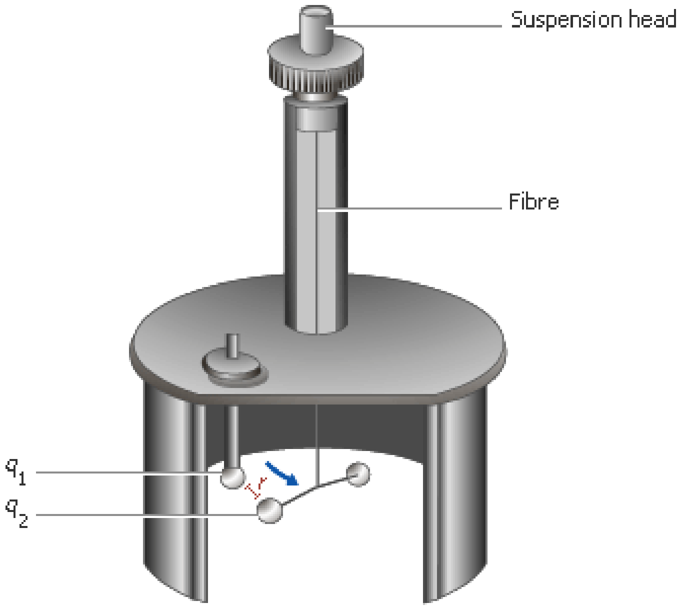
Coulomb's law

- The experimental law of Coulomb (1785)
  - http://navercast.naver.com/contents.nhn?contents_id=4647

$$\mathbf{F} = \mathbf{a}_R k \frac{q_1 q_2}{r^2} \qquad k = 9 \times 10^9 \ \mathrm{N{\bullet}m^2/C^2}$$

$$2.0 \pm 10^{-15}$$





Electrostatics in Free Space

- Electric field density : the force per unit charge (very small)

$$\mathbf{E} = \lim_{q \to 0} \frac{\mathbf{F}}{q} \; (\mathrm{V/m})$$

Electrostatics in Free Space

- The two fundamental postulates of electrostatics in free space.

$$\nabla \cdot \mathbf{E} = \frac{\rho_\upsilon}{\varepsilon_0} \Rightarrow \int_V \nabla \cdot \mathbf{E} d\upsilon = \frac{1}{\varepsilon_0}\int_V \rho_\upsilon d\upsilon \rightarrow \oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\varepsilon_0}$$

← Gauss's law

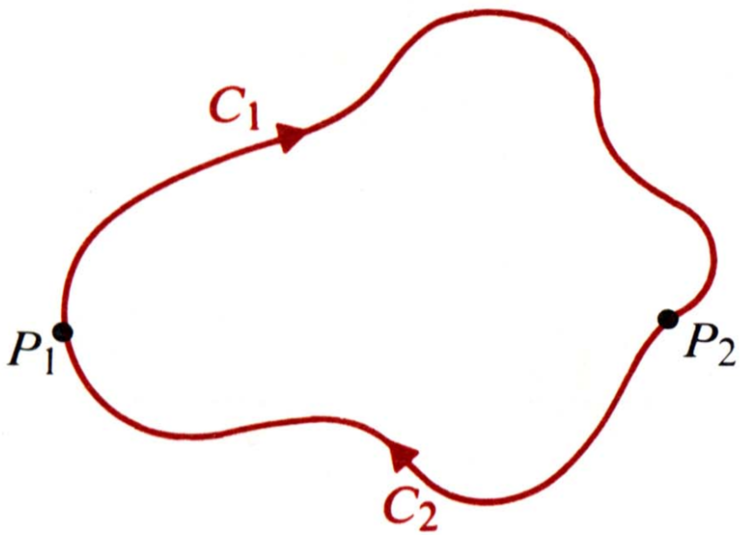
$$\nabla \times \mathbf{E} = 0 \Rightarrow \oint_C \mathbf{E} \cdot dl = 0$$

← Kirchhoff's voltage law

Static E is conservative !!

- Scalar line integral of E is independent of the path; it depends only on the end points.

$$\nabla \times \mathbf{E} = 0 \rightarrow \int_S (\nabla \times \mathbf{E}) \bullet d\mathbf{s} = \oint_C \mathbf{E} \bullet d\mathbf{l} = 0$$



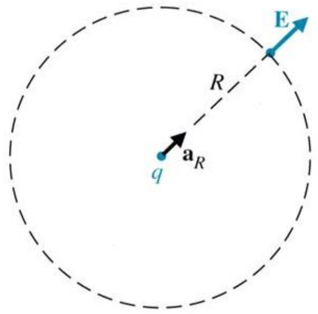
$$\int_{C_1} \mathbf{E} \bullet d\mathbf{l} + \int_{C_2} \mathbf{E} \bullet d\mathbf{l} = 0$$

$$\int_{P_1 C_1}^{P_2} \mathbf{E} \bullet d\mathbf{l} = -\int_{P_2 C_2}^{P_1} \mathbf{E} \bullet d\mathbf{l}$$

$$\int_{P_1 C_1}^{P_2} \mathbf{E} \bullet d\mathbf{l} = \int_{P_1 C_2}^{P_2} \mathbf{E} \bullet d\mathbf{l} \quad !!$$

Coulomb's Law

- Point charge at the origin



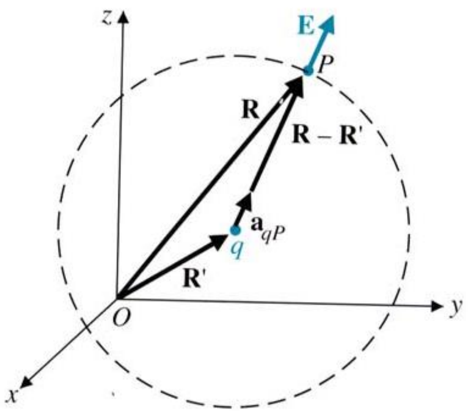
(a) Point charge at the origin.

$$\oint_S \mathbf{E} \bullet d\mathbf{s} = \oint_S (E\mathbf{a}_\mathbf{R}) \bullet \mathbf{a}_\mathbf{R} ds = \frac{q}{\varepsilon_0}$$

$$E_R \oint_S ds = E_R 4\pi R^2 = \frac{q}{\varepsilon_0}$$

$$\mathbf{E} = \mathbf{a}_\mathbf{R} E_R = \mathbf{a}_\mathbf{R} \frac{q}{4\pi\varepsilon_0 R^2} \, (\mathrm{V/m})$$

- Point charge not at the origin



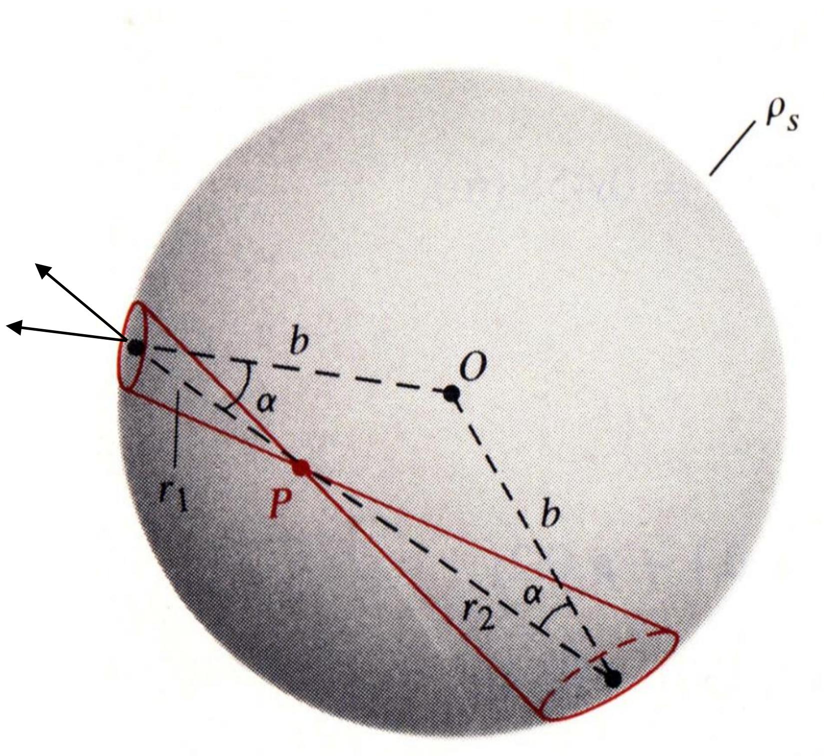
(b) Point charge not at the origin.

$$\mathbf{E}_p = \mathbf{a}_{qp} \frac{q}{4\pi\varepsilon_0 |\mathbf{R} - \mathbf{R}'|^2} \, (\mathrm{V/m}), \quad \mathbf{a}_{qp} = \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|}$$

$$\mathbf{E}_p = \frac{q(\mathbf{R} - \mathbf{R}')}{4\pi\varepsilon_0 |\mathbf{R} - \mathbf{R}'|^3}$$

Coulomb's Law

- Example 3.2
  - A total charge Q is put on a thin spherical shell of radius b, show that **E** inside the shell is zero.



$$\rho_s = \frac{Q}{4\pi b^2}$$

$$dE = \frac{\rho_s}{4\pi\varepsilon_0}\left(\frac{ds_1}{r_1^2} - \frac{ds_2}{r_2^2}\right)$$

$$d\Omega = \frac{ds_1}{r_1^2}\cos\alpha = \frac{ds_2}{r_2^2}\cos\alpha$$

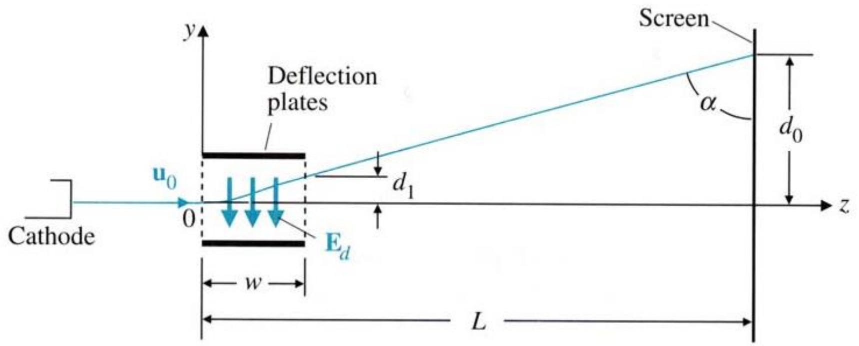
$$dE = \frac{\rho_s}{4\pi\varepsilon_0}\left(\frac{d\Omega}{\cos\alpha} - \frac{d\Omega}{\cos\alpha}\right) = 0$$

Coulomb's Law

- When a point charge q_2 is placed in the field of another point charge q_1, a force $\boldsymbol{F}_{12}$ is experienced by q_2 due to electric field $\boldsymbol{E}_{12}$ of q_1 at q_2

$$\mathbf{F}_{12} = q_2\mathbf{E}_{12} = \mathbf{a}_{12}\frac{q_1 q_2}{4\pi\varepsilon_0 R_{12}^2}\left(\mathrm{N}\right)$$

- Example 3.3 : *Find the vertical deflection of the electrons on the fluorescent screen at z=L*



$$d_0 = d_1 + d_2 = \frac{eE_d}{mu_0^2}w\left(L - \frac{w}{2}\right)$$

Electric Field due to Discrete Charges

- Electric field intensity $\mathbf{a}_R q / R^2$ is a linear function.
- The principle of superposition applies and the total electric field at a point is the vector sum of the fields caused by all the individual charges.

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1(\mathbf{R}-\mathbf{R}_1')}{|\mathbf{R}-\mathbf{R}_1'|^3} + \frac{q_2(\mathbf{R}-\mathbf{R}_2')}{|\mathbf{R}-\mathbf{R}_2'|^3} + \ldots + \frac{q_1(\mathbf{R}-\mathbf{R}_n')}{|\mathbf{R}-\mathbf{R}_n'|^3}\right]$$

or

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{n}\frac{q_k(\mathbf{R}-\mathbf{R}_k')}{|\mathbf{R}-\mathbf{R}_k'|^3} \quad (\text{V/m})$$

Electric Field due to Discrete Charges

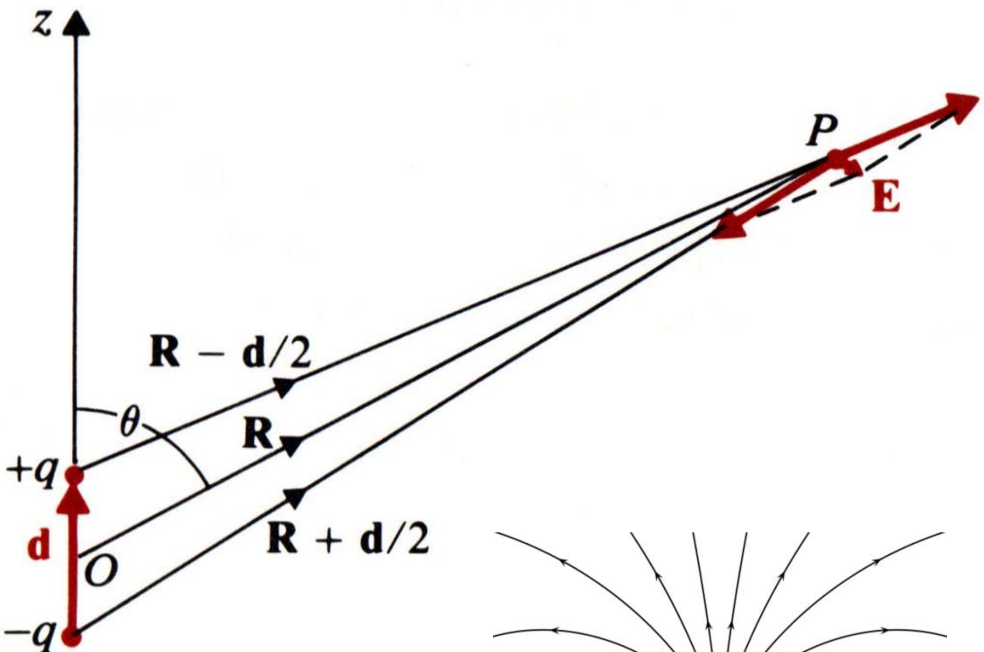
- Electric dipole
  - Dipole moment, $p = qd$

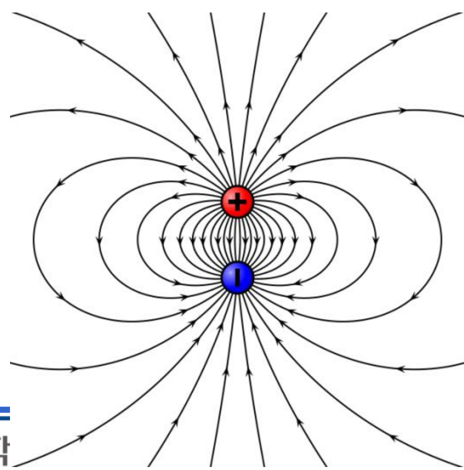
$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\left\{\frac{\mathbf{R}-\frac{\mathbf{d}}{2}}{\left|\mathbf{R}-\frac{\mathbf{d}}{2}\right|^3} - \frac{\mathbf{R}+\frac{\mathbf{d}}{2}}{\left|\mathbf{R}+\frac{\mathbf{d}}{2}\right|^3}\right\}$$

$$\mathbf{p} = q\mathbf{d} = p\mathbf{a}_z, \quad R \gg d$$

$$\mathbf{E} = \frac{p}{4\pi\varepsilon_0 R^3}\left(\mathbf{a}_R 2\cos\theta + \mathbf{a}_\theta \sin\theta\right) \quad (\mathrm{V/m})$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0 R^3}\left(3\frac{\mathbf{R}\cdot\mathbf{p}}{R^2}\mathbf{R} - \mathbf{p}\right) \quad (\mathrm{V/m})$$





Electric Field due to a Continuous Distribution

- The electric field caused by a continuous distribution of charge can be obtained by integrating (superposing) the contribution of an element of charge over the charge distribution.

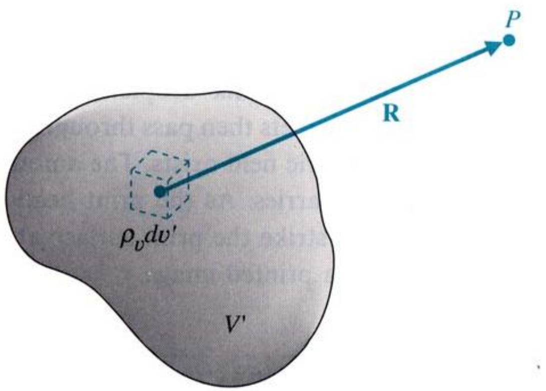


FIGURE 3-3 Electric field due to a continuous charge distribution.

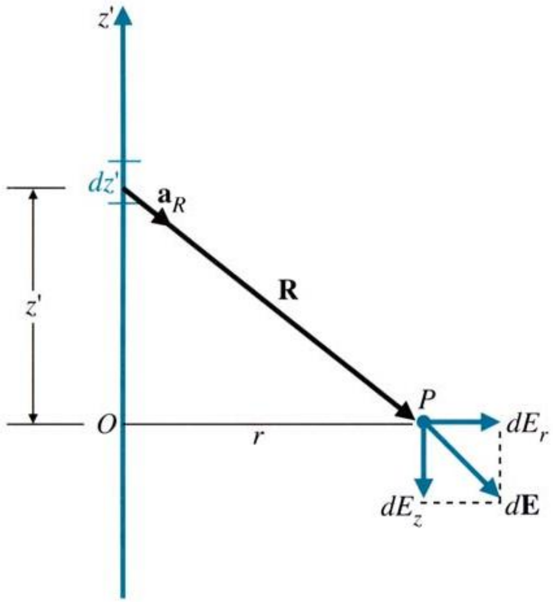
$$d\mathbf{E} = \mathbf{a}_\mathbf{R} \frac{\rho_\upsilon d\upsilon'}{4\pi\varepsilon_0 R^2} \left(\mathrm{V/m}\right)$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int_{V'} \mathbf{a}_\mathbf{R} \frac{\rho_\upsilon d\upsilon'}{R^2} \left(\mathrm{V/m}\right)$$

- Surface charge $\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int_{S'} \mathbf{a}_\mathbf{R} \frac{\rho_s ds'}{R^2} \left(\mathrm{V/m}\right)$
- Line charge $\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int_{L'} \mathbf{a}_\mathbf{R} \frac{\rho_l dl'}{R^2} \left(\mathrm{V/m}\right)$

Electric Field due to a Continuous Distribution

- Example 3.4



$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_{L'} \rho_l \frac{\mathbf{R}}{R^3} dl' \quad (\mathrm{V/m})$$

$$\mathbf{R} = \mathbf{a}_r r - \mathbf{a}_z z', dl' = \mathbf{a}_z dz'$$

$$d\mathbf{E} = \frac{\rho_l dz'}{4\pi\varepsilon_0} \frac{\mathbf{a}_r r - \mathbf{a}_z z'}{\left(r^2 + z^2\right)^{3/2}} = \mathbf{a}_r dE_r + \mathbf{a}_z dE_z$$

$$d\mathbf{E} = \mathbf{a}_r dE_r = \mathbf{a}_r \frac{\rho_l r}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{dz'}{\left(r^2 + z^2\right)^{3/2}}$$

or

$$\mathbf{E} = \mathbf{a}_r \frac{\rho_l}{2\pi\varepsilon_0 r} (\mathrm{V/m})$$

Gauss's Law and applications

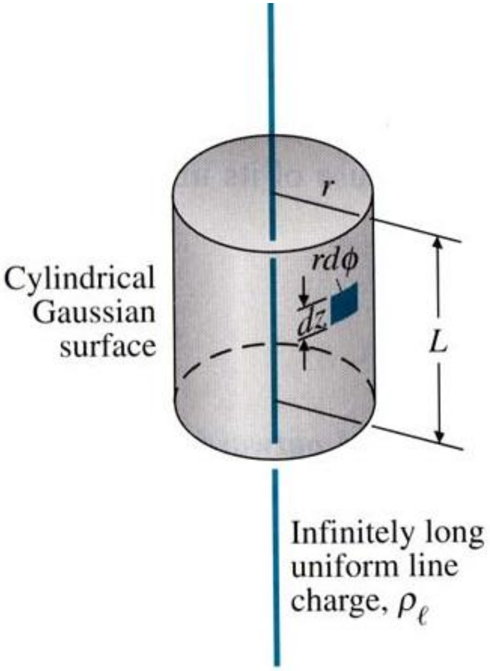
- Gauss's law
  - The total outward flux of the E field over any closed surface in free space is equal to the total charge enclosed in the surface divided by ε_0

$$\oint_S \mathbf{E} \bullet d\mathbf{s} = \frac{Q}{\varepsilon_0}$$

  - Useful in determining the **E** field of charge distributions with some symmetry conditions

Gauss's Law and applications

- Infinitely long line charge (example 3-5)



$$d\mathbf{s} = \mathbf{a}_r r\,d\phi\,dz$$

$$\oint_S \mathbf{E} \bullet d\mathbf{s} = \int_0^L \int_0^{2\pi} E_r r\,d\phi\,dz = 2\pi r L E_r$$

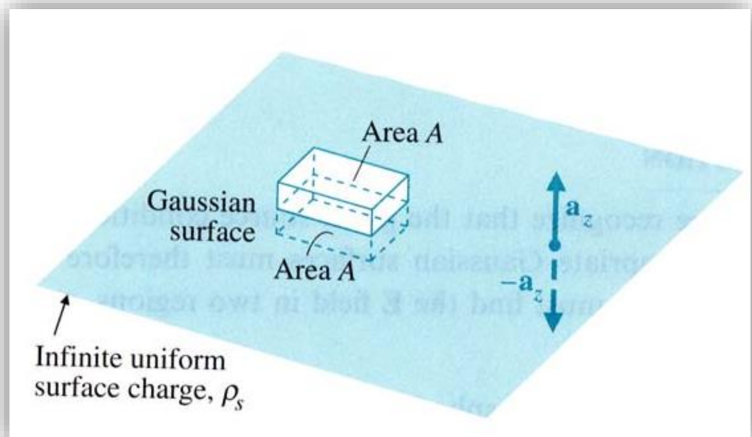
$$Q = \rho_l L$$

$$2\pi r L E_r = \frac{\rho_l L}{\varepsilon_0} \rightarrow \mathbf{E} = \mathbf{a}_r \frac{\rho_l}{2\pi\varepsilon_0 r}$$

What if the line charge is of a finite length ??

Gauss's Law and applications

- Infinite planar charge (example 3-6)



On the top face,

$$\mathbf{E}\cdot d\mathbf{s} = (\mathbf{a}_z E_z)\cdot(\mathbf{a}_z ds) = E_z ds$$

On the bottom face,

$$\mathbf{E}\cdot d\mathbf{s} = (-\mathbf{a}_z E_z)\cdot(-\mathbf{a}_z ds) = E_z ds$$

$$\oint_S \mathbf{E}\cdot d\mathbf{s} = 2E_z \int_A ds = 2E_z A$$

total charge enclosed in the box : $Q = \rho_s A$

$$2E_z A = \rho_s A$$

$$\mathbf{E} = \mathbf{a}_z E_z = \mathbf{a}_z \frac{\rho_s}{2\varepsilon_0}, \quad z>0$$

$$\mathbf{E} = -\mathbf{a}_z E_z = -\mathbf{a}_z \frac{\rho_s}{2\varepsilon_0}, \quad z<0$$

- Comparing lighting schemes

Gauss's Law and applications

- Spherical electron cloud (example 3.7)

$a)\ \ 0 \le R \le b$

$\mathbf{E}=\mathbf{a}_R E_R,\quad d\mathbf{s}=\mathbf{a}_R ds \rightarrow$ The total outward E flux is

$$\oint_{S'} \mathbf{E} \bullet d\mathbf{s} = E_R \int_{S_i} ds = E_R 4\pi R^2$$

The total charge enclosed within the Gaussian surface is

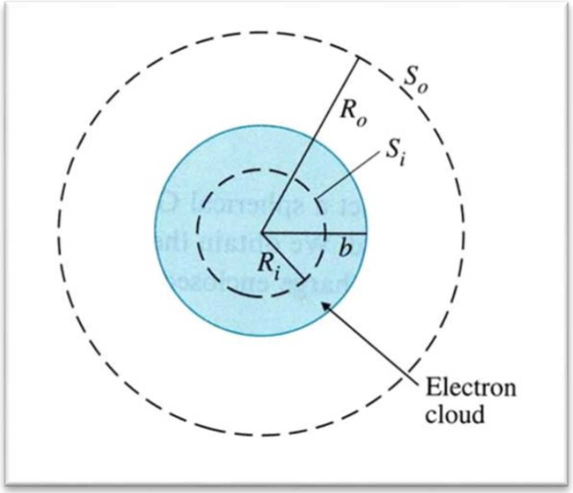
$$Q = \int_V \rho_\upsilon d\upsilon = -\rho_0 \int_V d\upsilon = -\rho_0 \frac{4\pi}{3} R^3$$

$$\mathbf{E} = -\mathbf{a}_R \frac{\rho_0}{3\varepsilon_0} R, \quad 0 \le R \le b$$

$b)\ R \ge b$

$$Q = -\rho_0 \frac{4\pi}{3} b^3$$

$$\mathbf{E} = -\mathbf{a}_R \frac{\rho_0 b^3}{3\varepsilon_0 R^2}$$



$\rho_\upsilon = -\rho_0$ for $0 \le R \le b$

$\rho_\upsilon = 0$ for $R > b$

- Spherical shell ?
- Gauss' law and ionization energy in atoms

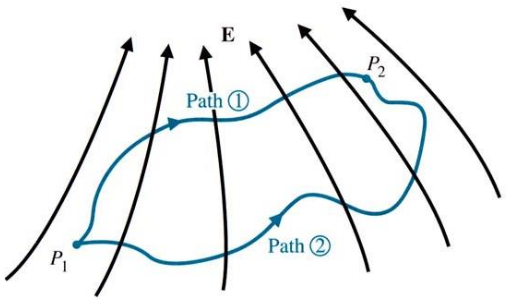
Electrical potential

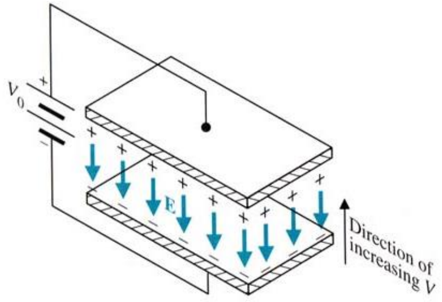
- From the null identity, $\nabla \times \mathbf{E} = 0 \rightarrow \mathbf{E} = -\nabla V$
- Scalar quantities are easy to handle than vector quantities.
- If we can determine V more easily, then E can be found by a gradient operation.

- Work done from point P_1 to point P_2

$$\frac{W}{q} = -\int_{P_1}^{P_2} \mathbf{E} \bullet d\mathbf{l} \quad (\text{J/C or V})$$

$$V_2 - V_1 = -\int_{P_1}^{P_2} \mathbf{E} \bullet d\mathbf{l} \quad (\text{V})$$





- The meaning of gradient
- Direction of gradient & equipotential surface

Electrical potential

- The electric potential of a point at a distance *R* from a point charge *q* referred to that at infinity

$$V = -\int_{\infty}^{R} \mathbf{a}_R \left(\frac{q}{4\pi\varepsilon_0 R^2} \right) \bullet (\mathbf{a}_R dR) \rightarrow V = \frac{q}{4\pi\varepsilon_0 R}$$

- The potential difference between P_1 and P_2 at distances R_2 and R_1, respectively, from *q* is

$$V_{21} = V_{P_2} - V_{P_1} = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{R_2} - \frac{1}{R_1} \right)$$

- The electric potential at R due to a system of n discrete charges, $q_1, q_2, \ldots, q_n$ located at $\mathbf{R}'_1, \mathbf{R}'_2, \ldots, \mathbf{R}'_n$

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{n} \frac{q_k}{\left| \mathbf{R} - \mathbf{R}'_k \right|} \qquad (\mathrm{V})$$

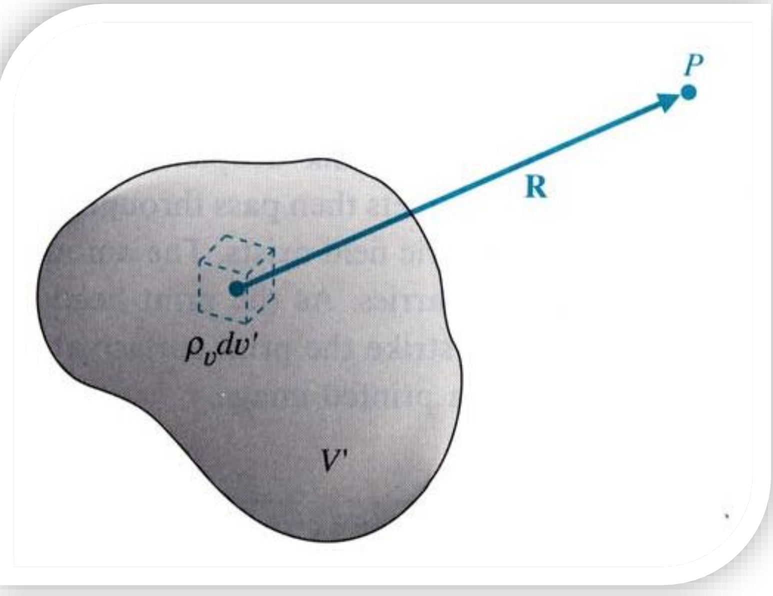
Electrical potential

- Continuous charge distributions

$$V = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho_\upsilon}{R}\, d\upsilon'$$

- For surface charge distribution

$$V = \frac{1}{4\pi\varepsilon_0}\int_{S'} \frac{\rho_s}{R}\, ds'$$

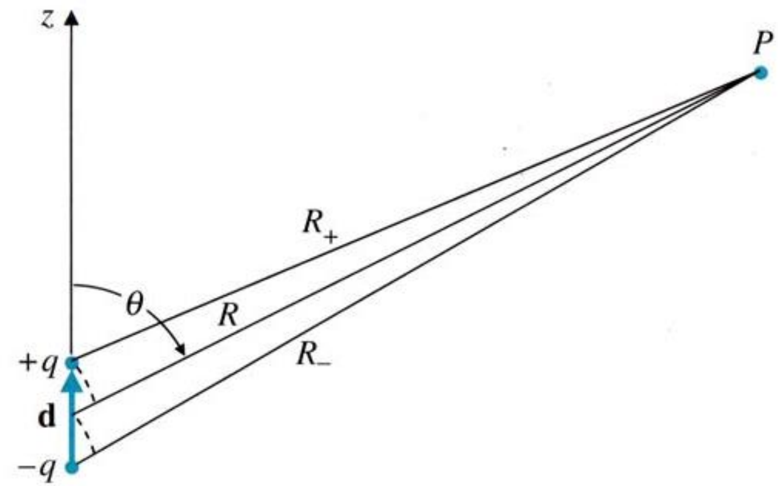


- For a line charge.

$$V = \frac{1}{4\pi\varepsilon_0}\int_{L'} \frac{\rho_l}{R}\, dl'$$

Electrical potential

- Electric dipole



$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_+} - \frac{1}{R_-}\right)$$

$$d << R \rightarrow R_+ = \left(R - \frac{d}{2}\cos\theta\right), R_- = \left(R + \frac{d}{2}\cos\theta\right)$$

Electrical potential

- Electric potential of electric dipole

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R - \frac{d}{2}\cos\theta} - \frac{1}{R + \frac{d}{2}\cos\theta}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{d\cos\theta}{R^2 - \frac{d^2}{4}\cos^2\theta}\right) \approx \frac{qd\cos\theta}{4\pi\varepsilon_0 R^2}$$

$$V = \frac{\mathbf{p}\cdot\mathbf{a_R}}{4\pi\varepsilon_0 R^2} \ \text{(V)} \ \textit{where } \mathbf{p} = q\mathbf{d} \text{ is the electric dipole moment}$$

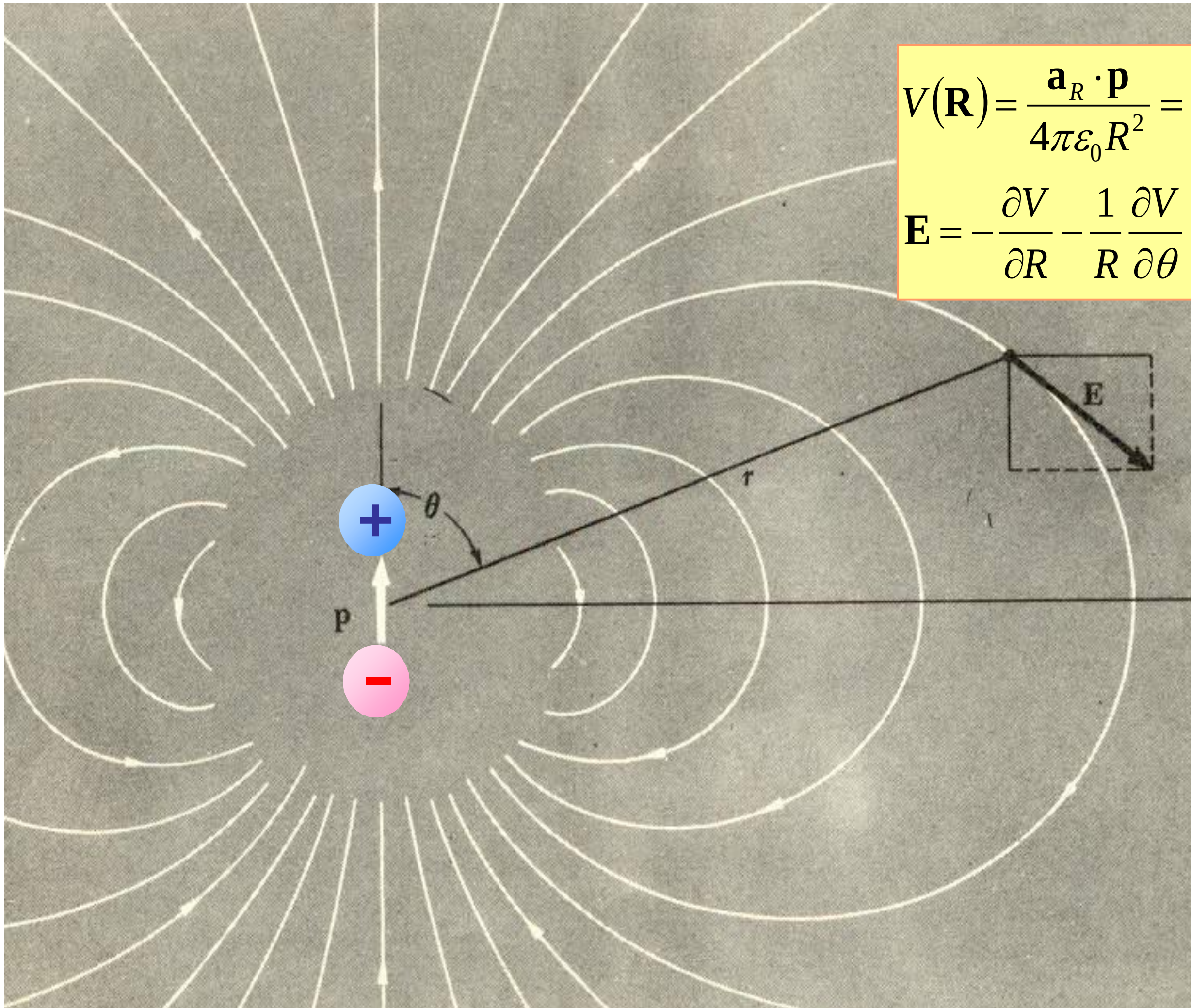
- Electric field

$$\mathbf{E} = -\nabla V = -\mathbf{a_R}\frac{\partial V}{\partial R} - \mathbf{a_\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} = \frac{p}{4\pi\varepsilon_0 R^3}\left(2\cos\theta\,\mathbf{a_R} + \sin\theta\,\mathbf{a_\theta}\right)$$

Note that both V and $\mathbf{E}$ are independent of ϕ, as expected

Electric field by a dipole moment

- Equipotential surface & E field line (Example 3-8)



$$V(\mathbf{R}) = \frac{\mathbf{a}_R \cdot \mathbf{p}}{4\pi\varepsilon_0 R^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{R^2}$$

$$\mathbf{E} = -\frac{\partial V}{\partial R} - \frac{1}{R}\frac{\partial V}{\partial \theta} = \frac{p}{4\pi\varepsilon_0 R^3}\left(2\cos\theta\,\mathbf{a}_R + \sin\theta\,\mathbf{a}_\theta\right)$$

$$d\mathbf{l} = k\mathbf{E}$$

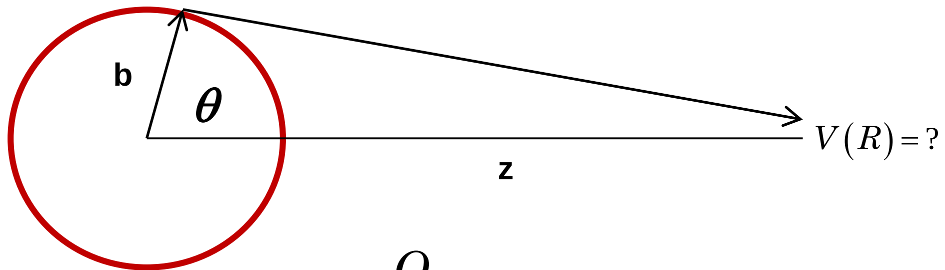
$$R = c_V\sqrt{\cos\theta}$$

$$R = c_E \sin^2\theta$$

Electric Potential

- Spherical Shell

$$V = \frac{Q}{4\pi\varepsilon_0 z}!!$$



$$\rho_s = \frac{Q}{4\pi b^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\rho_s b^2 \sin\theta d\theta d\phi}{\sqrt{z^2+b^2-2zb\cos\theta}} = \frac{\rho_s b^2}{2\varepsilon_0}\int_0^{\pi}\frac{\sin\theta d\theta}{\sqrt{z^2+b^2-2zb\cos\theta}}$$

$$= \frac{\rho_s b}{4\varepsilon_0 z}\left[\sqrt{z^2+b^2-2zb\cos\theta}\right]_0^{\pi} = \frac{\rho_s b^2}{\varepsilon_0 z} = \frac{Q}{4\pi\varepsilon_0 z}$$

Electric Potential

- Example 3.9

$$ds' = r'dr'd\phi' \quad \text{and } R = \sqrt{z^2 + r'^2}$$

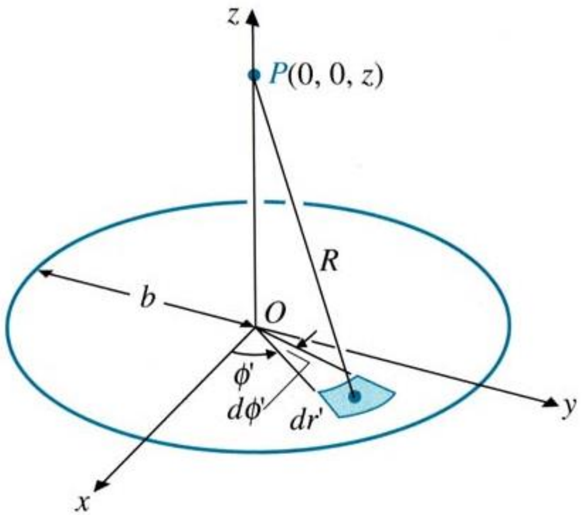
The electric potential at the point $P(0,0,z)$ referring to the point at infinity

$$V = \frac{\rho_s}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{b}\frac{r'}{\left(z^2 + r'^2\right)^{1/2}}dr'd\phi'$$

$$= \frac{\rho_s}{2\varepsilon_0}\left[\left(z^2 + b^2\right)^{1/2} - |z|\right]$$

$$\mathbf{E} = -\nabla V = -\mathbf{a}_z \frac{\partial V}{\partial z}$$

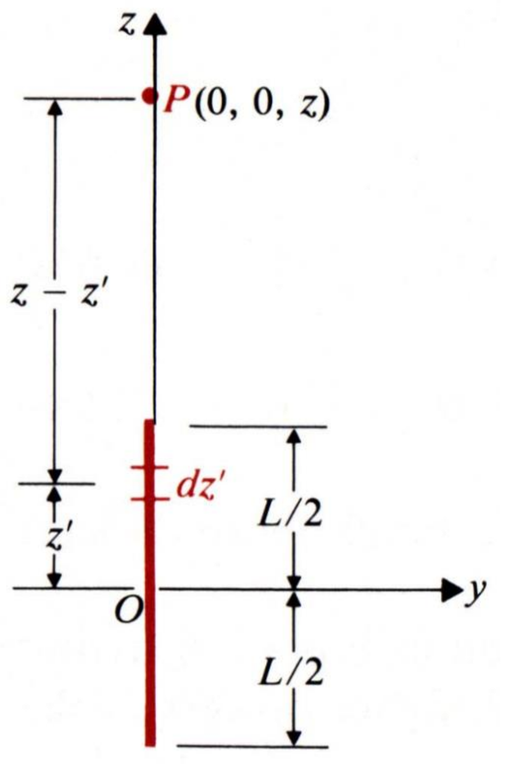
$$= \begin{cases} \mathbf{a}_z \dfrac{\rho_s}{2\varepsilon_0}\left[1 - z\left(z^2 + b^2\right)^{-1/2}\right], & z > 0 \\ -\mathbf{a}_z \dfrac{\rho_s}{2\varepsilon_0}\left[1 + z\left(z^2 + b^2\right)^{-1/2}\right], & z < 0 \end{cases}$$



- Spherical shell ?

Electric Potential

- Example 3.10



$$R = z - z', \quad z > L/2$$

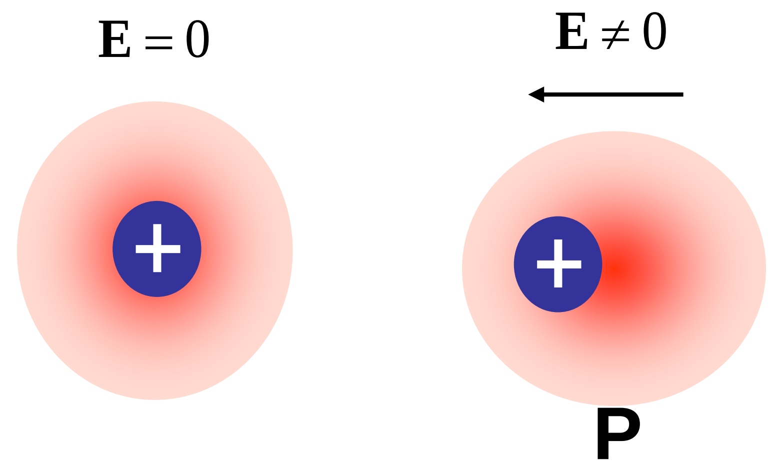
$$V = \frac{\rho_l}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{dz'}{z-z'} = \frac{\rho_l}{4\pi\varepsilon_0}\ln\left[\frac{z+L/2}{z-L/2}\right], \quad z > L/2$$

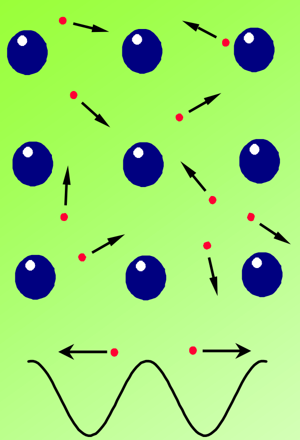
$$\mathbf{E} = -\nabla V$$

$$\mathbf{E} = -\mathbf{a}_z\frac{dV}{dz} = \mathbf{a}_z\frac{\rho_l L}{4\pi\varepsilon_0\left[z^2-(L/2)^2\right]}, \quad z > L/2$$

고체의 전기적 특성

free electrons(conductor)
bound electrons(insulator)



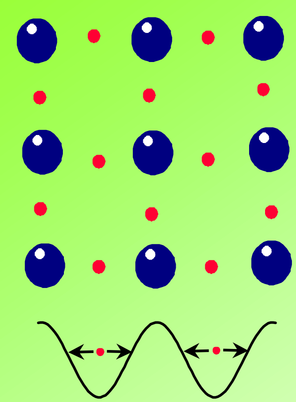


$$\mathbf{J}_{conduction}=\sigma\mathbf{E}$$

↑

$$\mathbf{F}=Q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$$

Conductor



Insulator

Dipole moment
P: polarization

$$\begin{cases}\rho_{polarization}=-\nabla\cdot\mathbf{P}\\ \mathbf{J}_{polarization}=\dfrac{\partial\mathbf{P}}{\partial t}\end{cases}$$

Conductors in static electric field

- **Inside a conductor (under static conditions)**

$$\rho_\upsilon = 0,\ \mathbf{E} = 0$$

- Boundary conditions at a conductor/free space interface

$$E_n = \frac{\rho_s}{\varepsilon_0} \leftarrow \oint_S \mathbf{E} \bullet d\mathbf{s} = E_n \Delta S = \frac{\rho_s \Delta S}{\varepsilon_0}$$

$$E_t = 0 \leftarrow \oint_{abcda} \mathbf{E} \bullet d\mathbf{l} = E_t \Delta w = 0$$

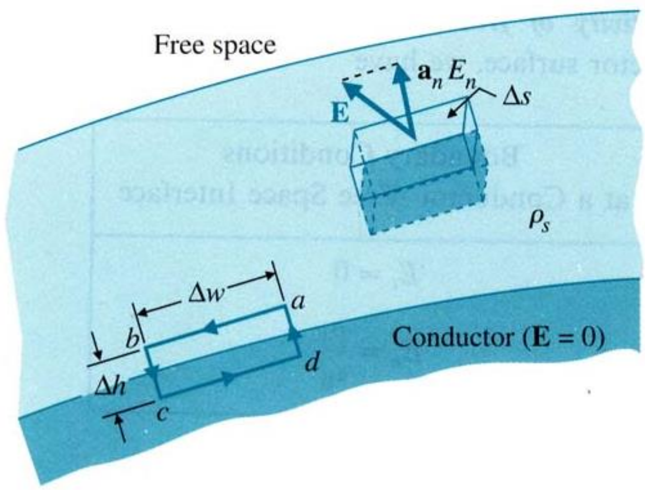


FIGURE 3-12 A conductor–free space interface.

- Shielding from outside electric fields

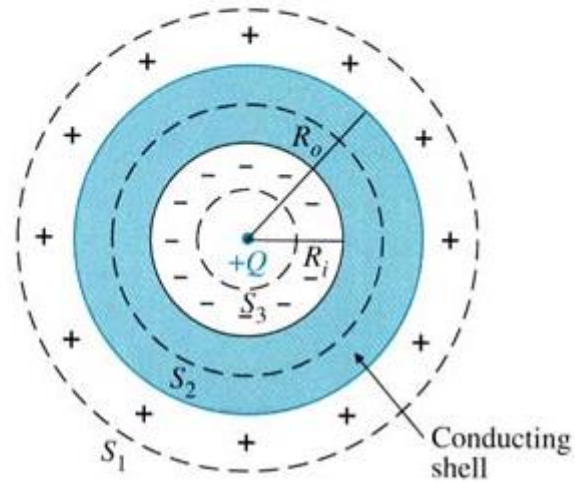
Conductors in static electric field

- Example 3.11

$$a)\ \ R > R_0 \quad \left(\text{Gaussian surface } S_1\right)$$

$$\oint_S \mathbf{E} \bullet d\mathbf{s} = E_{R1} 4\pi R^2 = \frac{Q}{\varepsilon_0} \rightarrow E_{R1} = \frac{Q}{4\pi\varepsilon_0 R^2}$$

$$V_{R1} = -\int_{\infty}^{R} E_{R1} dR = \frac{Q}{4\pi\varepsilon_0 R}$$



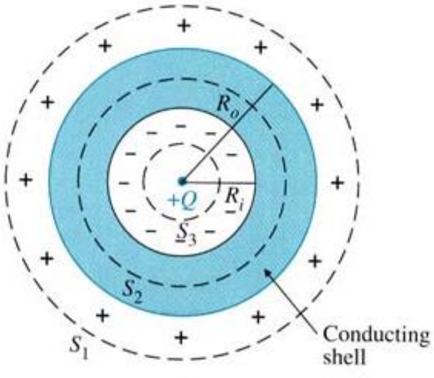
(a)

$$b)\ \ R_i < R < R_0 \ \left(\text{Gaussian surface } S_2\right)$$

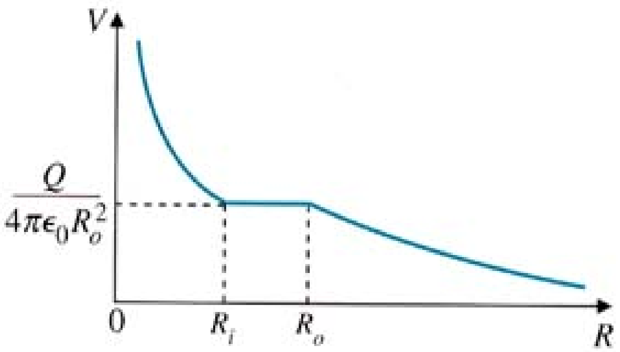
$$E_{R2} = 0, \leftarrow \mathbf{E} = 0 \text{ inside conductor}$$

$$V_2 = V_1\Big|_{R=R_0} = \frac{Q}{4\pi\varepsilon_0 R_0}$$

Conductors in static electric field



(a)



(c)

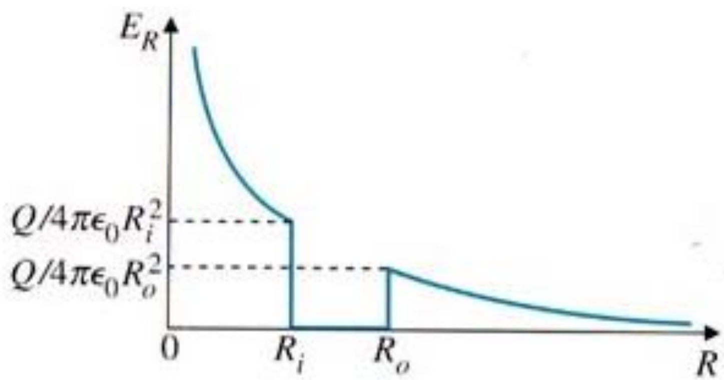
c) $R < R_i$

$$E_{R_3} = \frac{Q}{4\pi\varepsilon_0 R^2}$$

$$V_3 = -\int E_{R_3} dR + K = \frac{Q}{4\pi\varepsilon_0 R} + K$$

$$V_3 = V_2\big|_{R=R_i} \rightarrow K = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_0} - \frac{1}{R_i}\right)$$

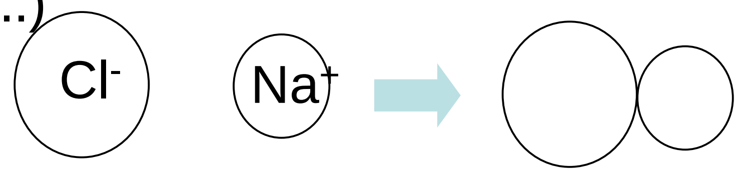
$$V_3 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} + \frac{1}{R_0} - \frac{1}{R_i}\right)$$



(b)

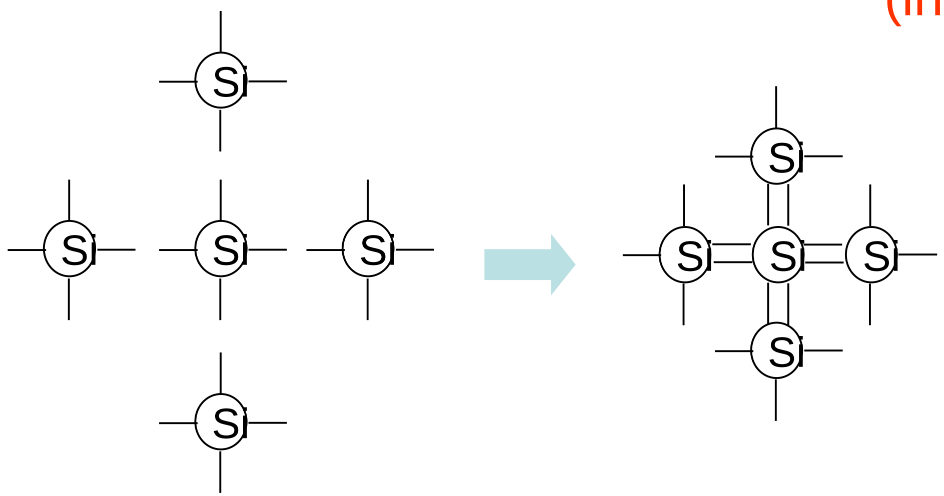
Atomic bonding in solid state materials

- Ionic bond (NaCl, ...)



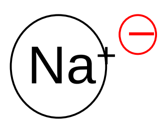
전자가 특정원자핵에 귀속
(insulator)

- Covalent bond(C, Si, Ge, ...)



전자가 주위의 몇 원자핵에
공유 (insulator, semiconductor)

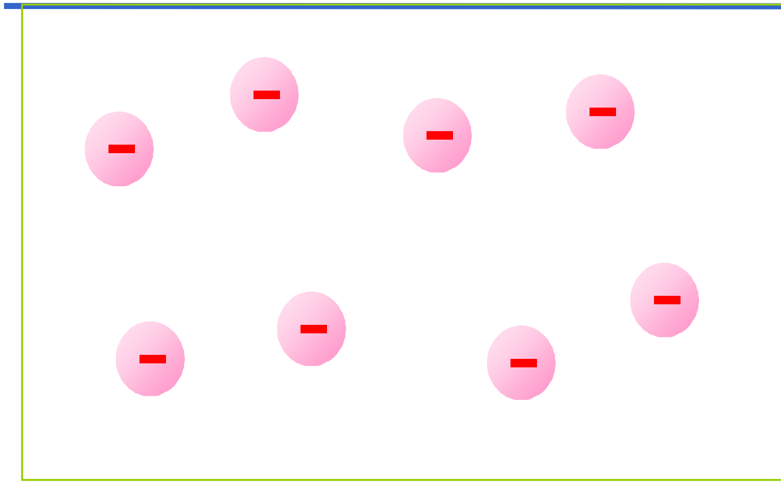
- Metallic bond(Na, K, ...)



전자가 모든 원자핵에
공유 (conductor) → Free electrons

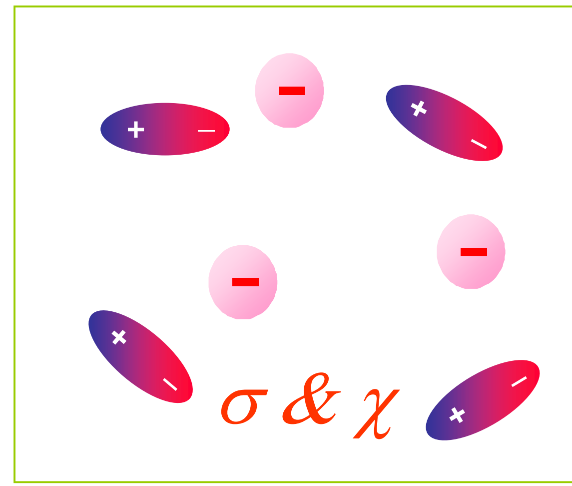
* Van der Waals bond(He, Ne, Ar, ...: inert gas): dipole interaction

도체(conductor) σ Free electron (Lorentz gas)

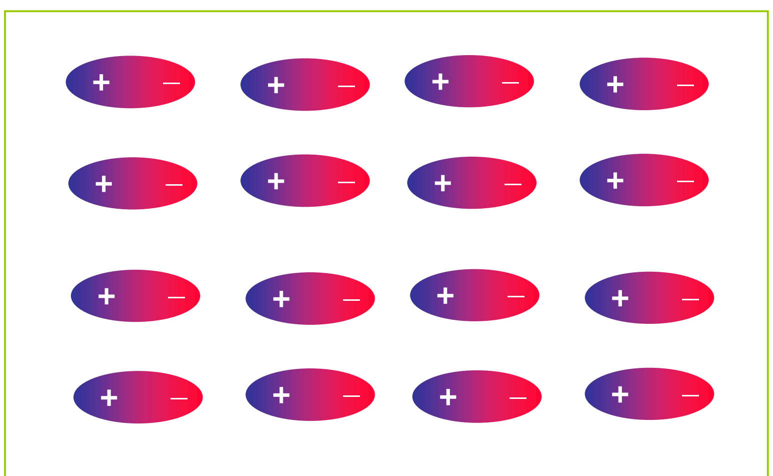


conductivity

$$\mathbf{J} = \sigma \mathbf{E}$$



부도체(insulator) χ

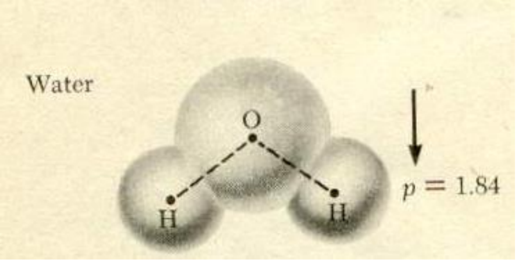


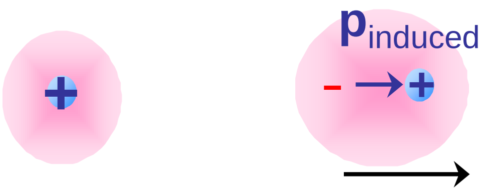
$$\mathbf{P} = \chi \mathbf{E}$$

susceptibility

Dielectrics in static E field

- Insulators (or dielectrics)
  - Bound charges
  - The induced electric dipoles will modify the electric field both inside and outside the dielectric material
  - Dielectric materials
    - Polar molecules
    - Nonpolar molecules

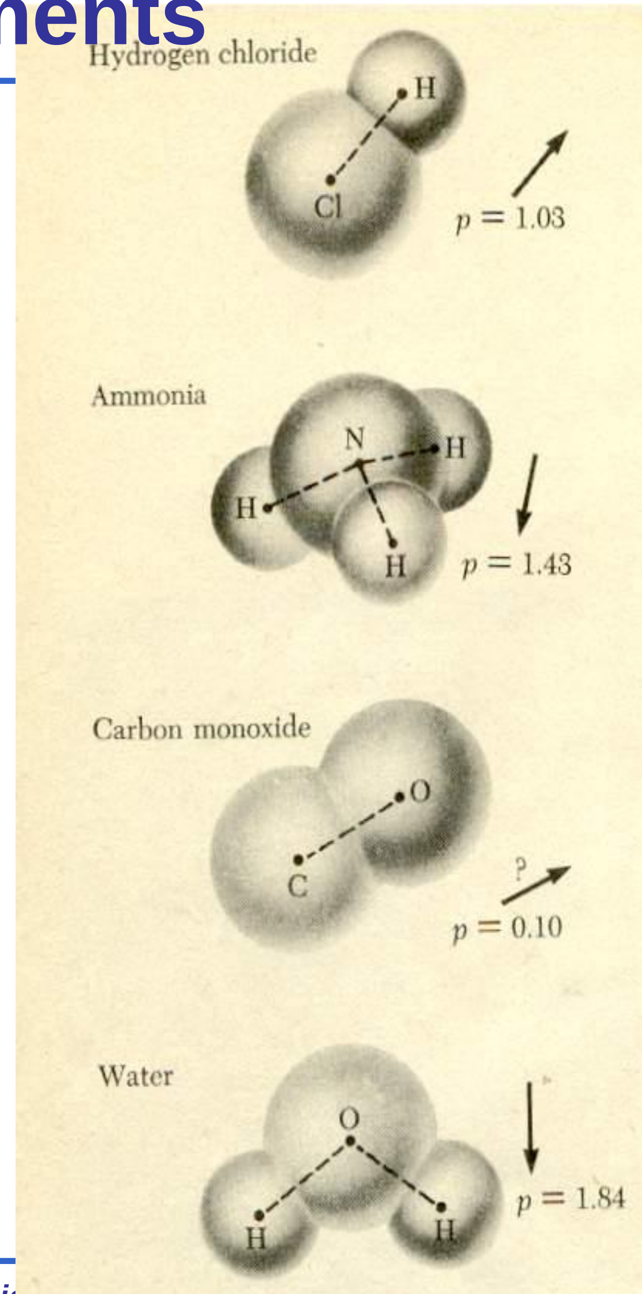




$\mathbf{E}_{applied} = 0$ $\mathbf{E}_{applied} \neq 0$

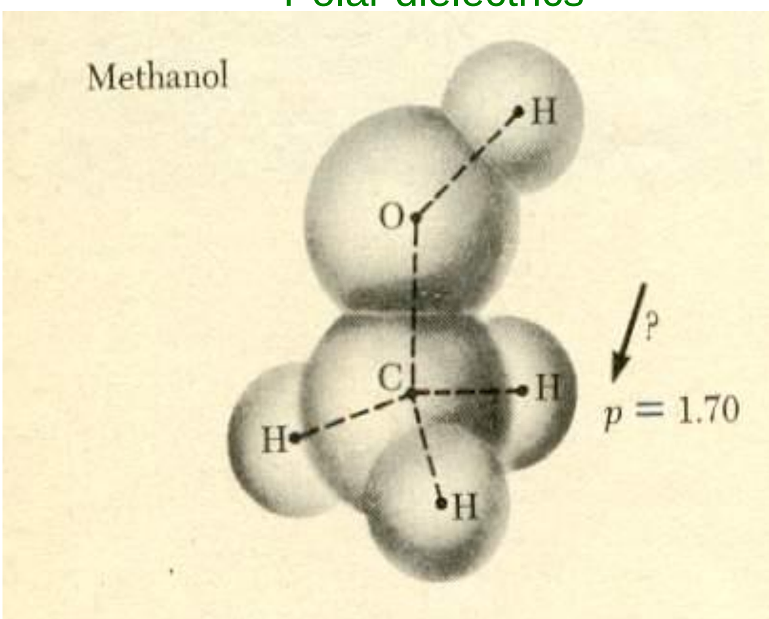
Two types of dipole moments

1. permanent dipole moment
2. induced dipole moment



permanent dipole moment

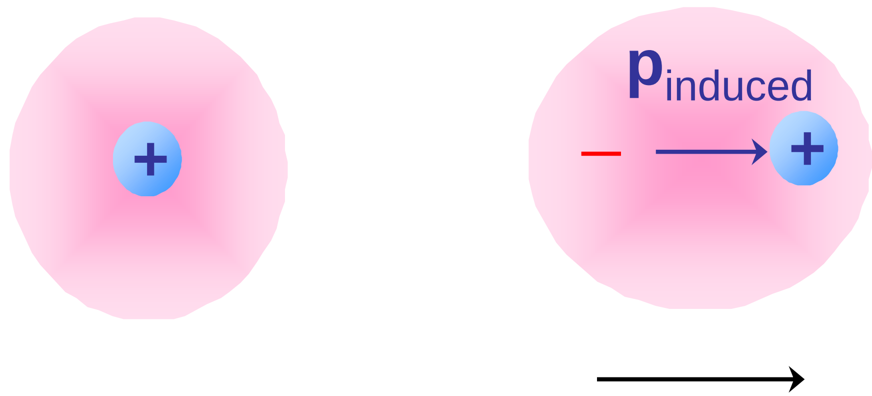
Polar dielectrics



Two types of dipole moments

- Induced dipole moment

Nonpolar dielectrics



$$\mathbf{E}_{applied} = 0 \qquad \mathbf{E}_{applied} \neq 0$$

$$\mathbf{p} = \alpha \mathbf{E}$$

$$\alpha : \text{ atomic polarizability}$$

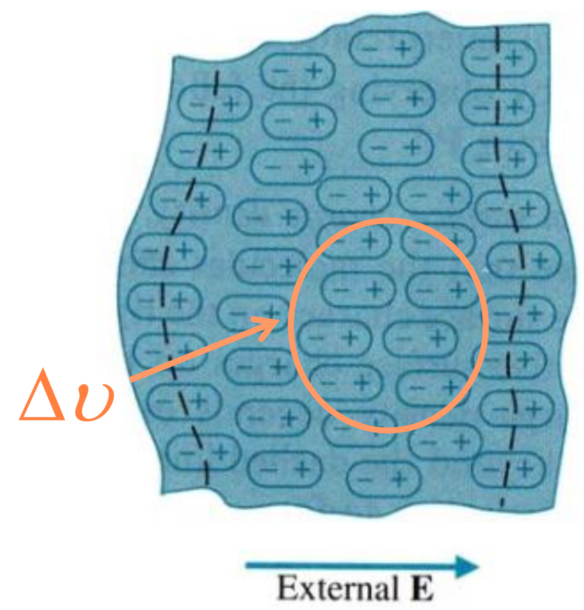
Static E field in dielectrics

- Polarization vector **P**
  - average volume density of dipole moment

$$\mathbf{P} = \lim_{\Delta\upsilon \to 0} \frac{\sum_{k=1}^{n\Delta\upsilon} \mathbf{p}_k}{\Delta\upsilon} \quad \left(\mathrm{C/m^2}\right) : \text{ volume density of electric dipole mement}$$

$$dV = \frac{\mathbf{P} \bullet \mathbf{a}_R}{4\pi\varepsilon_0 R^2} d\upsilon' \leftarrow d\mathbf{p} = \mathbf{P} d\upsilon'$$

$$V = \frac{1}{4\pi\varepsilon_0} \int_{V'} \frac{\mathbf{P} \bullet \mathbf{a}_R}{R^2} d\upsilon'$$



Mathematical proof of polarization charge

$$V=\frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\mathbf{P}\cdot\mathbf{a}_R}{R^2}d\upsilon'$$

$$R=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

$$\nabla'\left(\frac{1}{R}\right)=\frac{\mathbf{a}_R}{R^2}$$

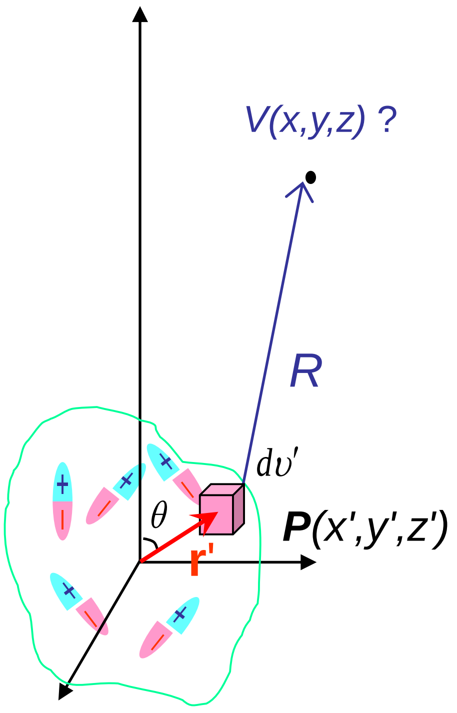
$$\nabla\cdot(f\mathbf{F})=f\nabla\cdot\mathbf{F}+\mathbf{F}\cdot\nabla f$$

$$\rightarrow\quad\frac{\mathbf{P}\cdot\mathbf{a}_R}{R^2}=\mathbf{P}\cdot\nabla'\left(\frac{1}{R}\right)=\nabla'\cdot\left(\frac{\mathbf{P}}{R}\right)-\frac{1}{R}\nabla'\cdot\mathbf{P}$$

divergence theorem

$$V(\mathbf{R})=\frac{1}{4\pi\varepsilon_0}\int_{V'}\nabla'\cdot\left(\frac{\mathbf{P}}{R}\right)d\upsilon'+\frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{(-\nabla'\cdot\mathbf{P})}{R}d\upsilon'$$

$$=\frac{1}{4\pi\varepsilon_0}\oint_{V'}\frac{\mathbf{P}\cdot\mathbf{n}}{R}ds'+\frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{(-\nabla'\cdot\mathbf{P})}{R}d\upsilon'$$



Dielectrics in static E field

$$V = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\mathbf{P}\cdot\mathbf{a}_R}{R^2} d\upsilon'$$

$$V = \frac{1}{4\pi\varepsilon_0}\oint_{S'} \frac{\mathbf{P}\cdot\mathbf{a}_n}{R} ds' + \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\left(-\nabla'\cdot\mathbf{P}\right)}{R} d\upsilon'$$

$$= \frac{1}{4\pi\varepsilon_0}\oint_{\dot{S}'} \frac{\rho_S}{R} ds' + \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho_\upsilon}{R} d\upsilon'$$

Polarization charge

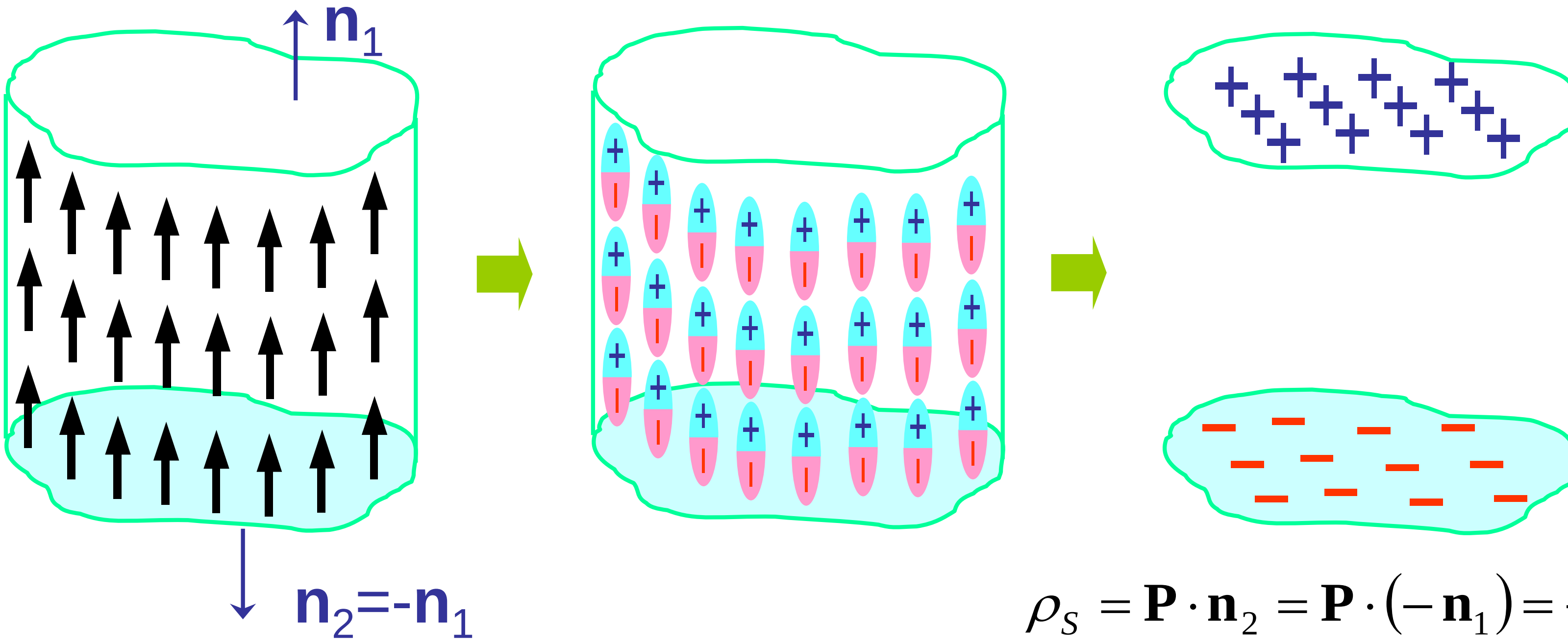
$$\begin{cases} \rho_S = \mathbf{P}\cdot\mathbf{n} \\ \rho_\upsilon = -\nabla\cdot\mathbf{P} \end{cases}$$

Physical meaning of polarization charge

$$\text{Polarization charge} \begin{cases} \rho_S = \mathbf{P}\cdot\mathbf{n} \\ \rho_\upsilon = -\nabla\cdot\mathbf{P} \end{cases}$$

when $\rho_\upsilon = -\nabla\cdot\mathbf{P} = 0$

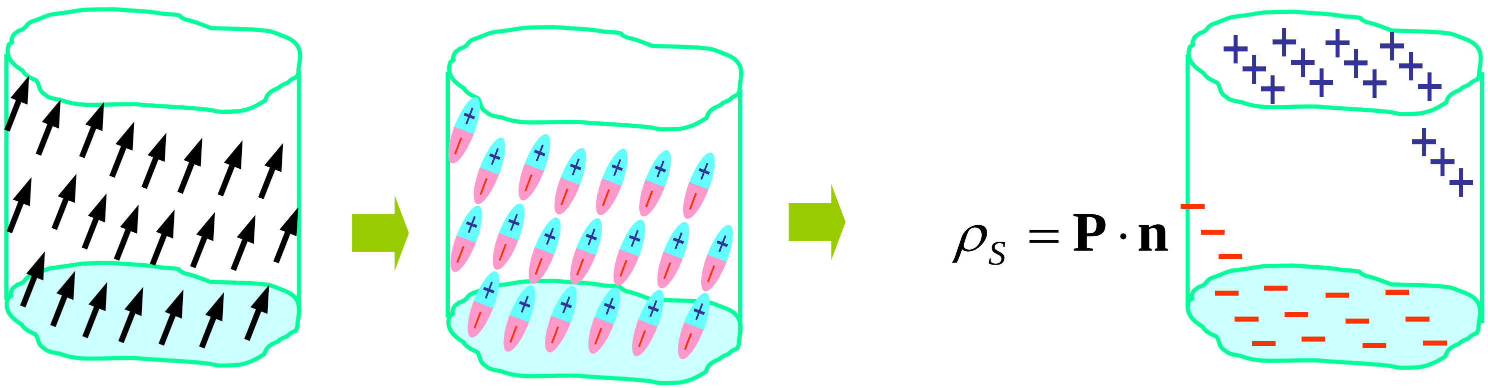
$$\rho_S = \mathbf{P}\cdot\mathbf{n}_1 = P$$



$$\rho_S = \mathbf{P}\cdot\mathbf{n}_2 = \mathbf{P}\cdot(-\mathbf{n}_1) = -P$$

Physical meaning of polarization charge

$$\rho_S = \mathbf{P}\cdot\mathbf{n}_1 = P_n$$

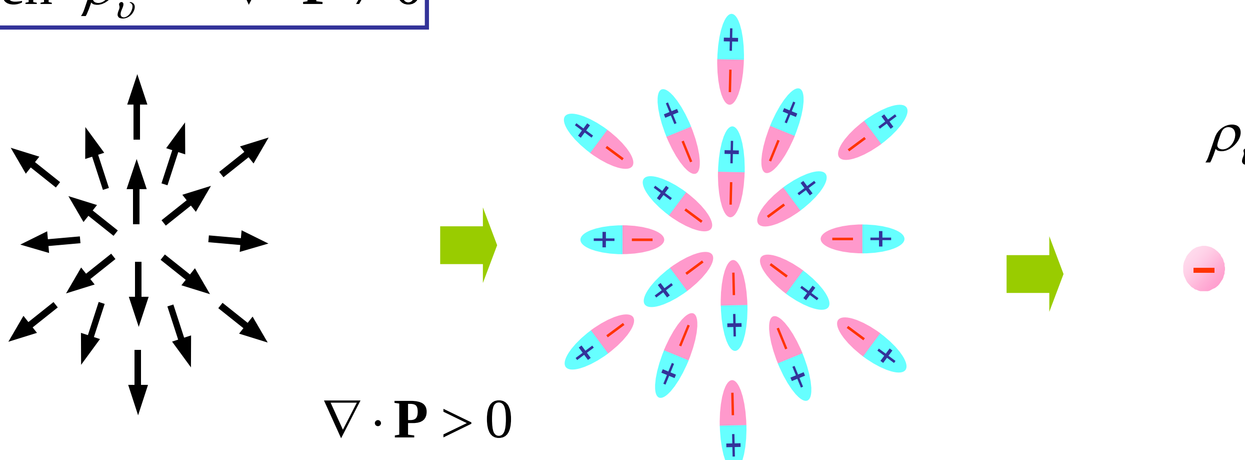


$$\rho_S = \mathbf{P}\cdot\mathbf{n}$$

$$\rho_S = \mathbf{P}\cdot\mathbf{n}$$

$$\rho_S = \mathbf{P}\cdot\mathbf{n}_2 = -P_n$$

when $\rho_\upsilon = -\nabla\cdot\mathbf{P} \neq 0$



$$\nabla\cdot\mathbf{P} > 0$$

$$\rho_\upsilon = -\nabla\cdot\mathbf{P}$$

Example

- The polarization vector in a dielectric sphere of radius R_0 is $\mathbf{P} = \mathbf{a}_z P_0$

- Determine

(a) the equivalent polarization surface and volume charge densities, and

$$\rho_{ps} = \mathbf{P}\cdot\mathbf{a}_R = P_0\left(\mathbf{a}_z\cdot\mathbf{a}_R\right) = P_0\cos\theta$$
$$\rho_{p\upsilon} = -\nabla\cdot\mathbf{P} = 0$$

(b) the total equivalent charge on the surface and inside of the sphere.

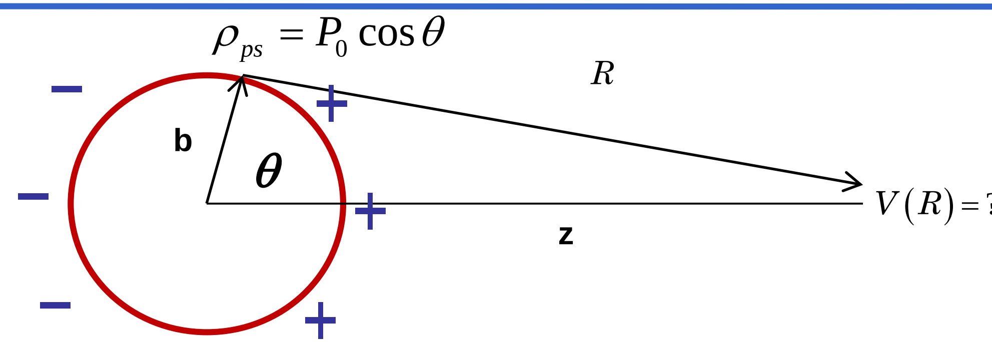
$$Q = \oint_C \rho_{ps}ds = \int_0^{2\pi}\int_0^{\pi} P_0\cos\theta R_0^2\sin\theta d\theta d\phi = 0$$

Thus, total charge on the sphere, $Q_s + Q_\upsilon = 0$

(c) Potential or electric field at z ?

Electric Potential

- Spherical Shell
  - V outside



$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\rho_{ps} b^2 \sin\theta d\theta d\phi}{\sqrt{z^2+b^2-2bz\cos\theta}} = \frac{P_0 b^2}{2\varepsilon_0}\int_0^{\pi}\frac{\cos\theta\sin\theta d\theta}{\sqrt{z^2+b^2-2zb\cos\theta}}$$

$\cos\theta = t$

$$\int_{-1}^{1}\frac{tdt}{\sqrt{z^2+b^2-2zbt}} = -\frac{\left(z^2+b^2+2zbt\right)\sqrt{z^2+b^2-2zbt}}{3b^2z^2}\Bigg|_{-1}^{1}$$

when $z > b$

$$V = \frac{P_0 b^3}{3\varepsilon_0 z^2} \rightarrow V = \frac{p}{4\pi\varepsilon_0 z^2} \qquad \leftarrow \mathbf{p}\left(\text{diploe moment}\right) = \frac{4}{3}\pi b^3 P_0\ \mathbf{a}_z$$

  - V inside the shell (z<b)? $V = \dfrac{P_0}{3\varepsilon_0} z$

Electric Flux Density and Dielectric Const

- Gauss's law inside dielectric with no surface charge

$$\text{Polarizati on charge,} \quad \rho_{\mathrm{p}} = -\nabla \cdot \mathbf{P}$$

$$\nabla \cdot \mathbf{E} = \frac{\left(\rho_\upsilon + \rho_{p\upsilon}\right)}{\varepsilon_0}$$

$$\rightarrow \nabla \cdot \mathbf{E} = \frac{\left(\rho_\upsilon - \nabla \cdot \mathbf{P}\right)}{\varepsilon_0}$$

$$\rightarrow \nabla \cdot \left(\varepsilon_0 \mathbf{E} + \mathbf{P}\right) = \rho_\upsilon$$

$$\Downarrow$$

$$\begin{cases} \mathbf{D} \equiv \varepsilon_0 \mathbf{E} + \mathbf{P} \quad \left(\mathrm{C/m^2}\right) \\ \nabla \cdot \mathbf{D} = \rho_{free} \quad \left(\mathrm{C/m^3}\right) \end{cases}$$

$$\oint_S \mathbf{D} \bullet d\mathbf{s} = Q \quad \text{: Generalized Gauss's law}$$

Electric Flux Density and Dielectric Const.

- Dielectric constant

electric susceptibility

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$$

$$\rightarrow \mathbf{D} = \varepsilon_0 \mathbf{E} + \chi_e \varepsilon_0 \mathbf{E} = \varepsilon_0 (1 + \chi_e) \mathbf{E}$$

$$\Downarrow$$

$$\begin{cases} \mathbf{D} = \varepsilon \mathbf{E} \\ \varepsilon = \varepsilon_R \varepsilon_0 \\ \varepsilon_R \equiv 1 + \chi_e \end{cases}$$

Permittivity (dielectric constant)

Relative permittivity (dielectric constant)

Common misunderstanding on E & D

오류 1 **E**를 인가했더니 $\chi_e\varepsilon_e$**P**가 유도 ?

induced polarization ? ⬅ *applied field ?* : not at all!

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$$

$$\mathbf{E} = \mathbf{E}_{\text{applied}} + \mathbf{E}_{\text{by dipoles}}$$

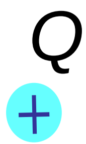
오류 2 **E**는 인가된 전기장, **D**는 유도된 전기장 ? : not at all!

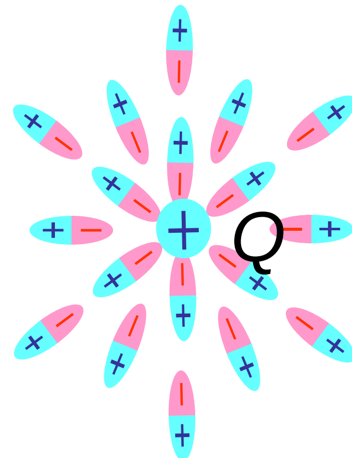
E와 **D**는 서로 다른 물리량.

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{D} = \mathbf{D}_{\text{applied}} + \mathbf{D}_{\text{by dipoles}}$$

Point charge in a dielectric





$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$E_2 = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$$

$$E_2 \le E_1 \quad \text{because} \quad \varepsilon \ge \varepsilon_0$$

➡ dielectric안에서 **E**-field는 약해짐

Dielectric constants

$\varepsilon_R = \text{costant} \geq 1$: dielectric materials

$\varepsilon_R = \varepsilon_R(\mathbf{E}) >> 1$: ferroelectric materials

Anisotropic materials:
dielectric tensor

$$\mathbf{D} = \overleftrightarrow{\varepsilon}\mathbf{E}$$

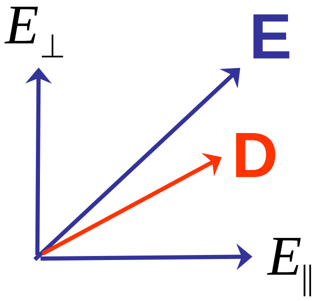
$$D_x = \varepsilon_{xx}E_x + \varepsilon_{xy}E_y + \varepsilon_{xz}E_z$$

$$D_y = \varepsilon_{yx}E_x + \varepsilon_{yy}E_y + \varepsilon_{yz}E_z$$

$$D_z = \varepsilon_{zx}E_x + \varepsilon_{zy}E_y + \varepsilon_{zz}E_z$$

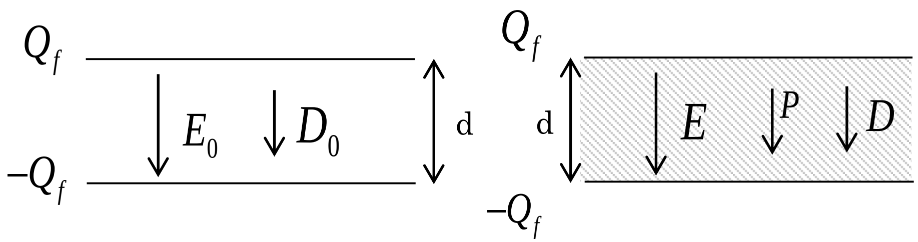
$$\overleftrightarrow{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

Material	ε_R
Air	1.0005
Paper	3
Ferrite(NiZn)	12.4
Glass	4-7
Water(distilled)	80
Barium titanate	1200



D & E Field in a parallel capacitor (필수!)

- Fixed charges



$$D_0 = \varepsilon_0 E_0 = \rho_{sf} = Q_f / A$$

$$D = D_0 = \rho_{sf} \text{ when } \rho_{sf} \text{ are kept const.}$$

$$E = D/\varepsilon = D_0/\varepsilon = \varepsilon_0 E_0 / \varepsilon_0 \varepsilon_R$$

$$\rightarrow E = E_0 / \varepsilon_R < E_0$$

$$V = \int_{+}^{-} \mathbf{E} \cdot \mathbf{ds} = Ed = E_0 d / \varepsilon_R = V_0 / \varepsilon_R$$

$$\rightarrow V < V_0$$

$$C = Q/V \rightarrow \therefore C = \varepsilon_R C_0$$

$$\rightarrow C > C_0$$

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \left(1 + \chi_e\right) \mathbf{E} = \varepsilon \mathbf{E}$$

$$\varepsilon = \varepsilon_R \varepsilon_0$$

$$P = \chi_e \varepsilon_0 E = (\varepsilon_R - 1)\varepsilon_0 E = \frac{(\varepsilon_R - 1)}{\varepsilon_R} \varepsilon_0 E_0$$

$$P = \varepsilon_0 (E_0 - E) \rightarrow E = E_0 - P/\varepsilon_0$$

$$D = \varepsilon_0 E + P = D_0 = \varepsilon_0 E_0$$

$$|\rho_{ps}| = \mathbf{P} \bullet \mathbf{n} = P = \left(\frac{\varepsilon_R - 1}{\varepsilon_R}\right) \varepsilon_0 E_0 = \left(\frac{\varepsilon_R - 1}{\varepsilon_R}\right) \rho_{sf}$$

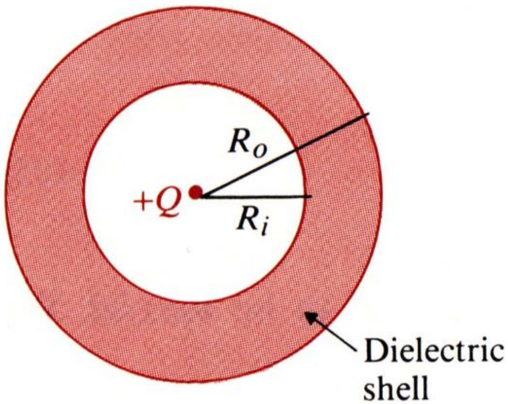
$$\rightarrow \rho_{ps} = -\rho_{sf} \left(\varepsilon_R = \infty\right)$$

$$E_b = \frac{|\rho_{ps}|}{\varepsilon_0} = \frac{P}{\varepsilon_0}$$

$$E = E_0 - E_b$$

Dielectric shell

- Example 3-12
  - Determine **E**, *V*, **D** and **P** as functions of *R*



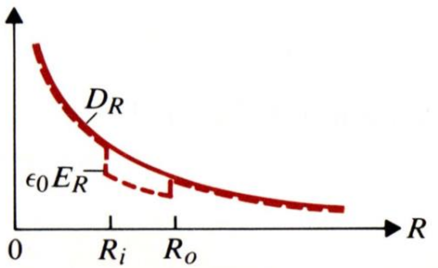
(a)

$$\mathbf{P} = \mathbf{D} - \varepsilon_0 \mathbf{E} = \varepsilon_0 (\varepsilon_r - 1) \mathbf{E}$$

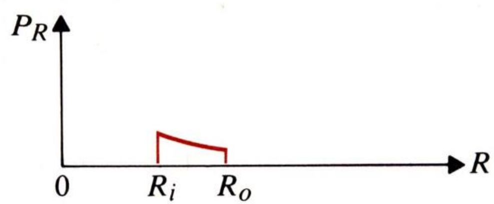
$$R_i < R < R_o$$

$$P_{R_2} = \varepsilon_0 (\varepsilon_r - 1) \mathbf{E}_{R_2} = \left(1 - \frac{1}{\varepsilon_r}\right) \frac{Q}{4\pi R^2}$$

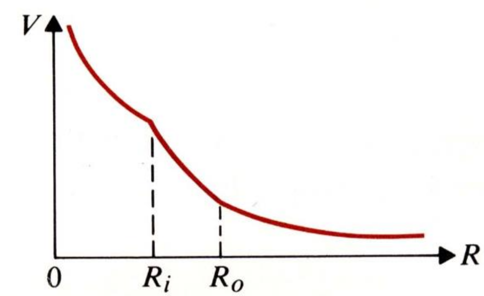
$$V \rightarrow continuous$$



(b)



(c)



(d)

Dielectric shell

- Dielectric charge : surface & volume

$$\rho_{ps}\left(\text{surface charge}\right) = \mathbf{P}\bullet\mathbf{a}_n\,? \quad \rho_v\left(\text{volume charge}\right) = -\nabla\bullet\mathbf{P}\,?$$

$$\rho_{ps}\Big|_{R=R_i} = \mathbf{P}\bullet\left(-\mathbf{a}_R\right)\Big|_{R=R_i} = -\left(1-\frac{1}{\varepsilon_r}\right)\frac{Q}{4\pi R_i^2}$$

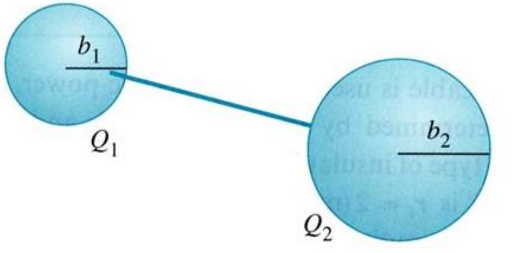
$$\rho_{ps}\Big|_{R=R_o} = \mathbf{P}\bullet\left(\mathbf{a}_R\right)\Big|_{R=R_o} = \left(1-\frac{1}{\varepsilon_r}\right)\frac{Q}{4\pi R_o^2}$$

$$\rho_{ps}\Big|_{R=R_i} \neq \rho_{ps}\Big|_{R=R_o}$$

$$\rho_p = -\nabla\bullet\mathbf{P} = -\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 P_{R_2}\right) = 0$$

Charge distribution

- Example 3.13
  - Electric field intensity at a conductor surface is higher at points of larger curvature.
  - a) the charges on the two spheres
  - b) the electric field intensities at the sphere surfaces



The spherical conductors at the same potential

$$\frac{Q_1}{4\pi\varepsilon_0 b_1} = \frac{Q_2}{4\pi\varepsilon_0 b_2} \rightarrow \frac{Q_1}{Q_2} = \frac{b_1}{b_2}$$

The charges on the spheres $\propto$ their radii

$$Q_1 + Q_2 = Q$$

$$Q_1 = \frac{b_1}{b_1 + b_2} Q \text{ and } Q_2 = \frac{b_2}{b_1 + b_2} Q$$

$$E = \frac{\rho_s}{\varepsilon_0}$$

The electric field intensities at the surfaces of the two conducting spheres

$$E_{1n} = \frac{Q_1}{4\pi\varepsilon_0 b_1^2} \text{ and } \frac{Q_2}{4\pi\varepsilon_0 b_2^2} \rightarrow \frac{E_{1n}}{E_{2n}} = \left(\frac{b_2}{b_1}\right)^2 \frac{Q_1}{Q_2} = \frac{b_2}{b_1}$$

Dielectric strength & Dielectric Constant

TABLE 3–1
Dielectric Constants and Dielectric Strengths of Some Common Materials

Material	Dielectric Constant	Dielectric Strength (V/m)
Air (atmospheric pressure)	1.0	3×10^6
Mineral oil	2.3	15×10^6
Paper	2–4	15×10^6
Polystyrene	2.6	20×10^6
Rubber	2.3–4.0	25×10^6
Glass	4–10	30×10^6
Mica	6.0	200×10^6

Dielectric strength

- Example 3.16 : Coaxial cable
  - Determine the inner radius r_i of the outer conductor so that the cable is to work at a voltage rating of 10 kV. (dielectric con. = 2.6 and dielectric strength of polystyrene =20×10^6 (V/m)
  -

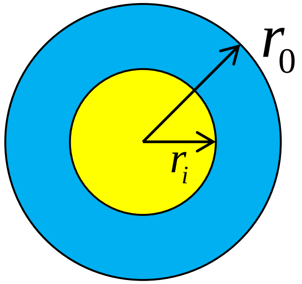
$$\mathbf{E} = \mathbf{a}_r E_r = \mathbf{a}_r \frac{\rho_l}{2\pi\varepsilon_r\varepsilon_0 r} \rightarrow V = -\int_{r_0}^{r_i} E_r dr = \frac{\rho_l}{2\pi\varepsilon_r\varepsilon_0 r} \ln\frac{r_0}{r_i}$$

$$10^4 = \frac{\rho_l}{2\pi(2.6)\varepsilon_0} \ln\frac{r_0}{r_i} \rightarrow \ln\frac{r_0}{r_i} = \left(\frac{5.2\pi\varepsilon_0}{\rho_l}\right)\times 10^4$$

$$\text{Maximum } E_r = 0.25\times\left(20\times10^6\right) = \frac{\rho_l}{2\pi(2.6)\varepsilon_0 r_i}$$

$$\rightarrow \left(\frac{\rho_l}{5.2\pi\varepsilon_0}\right) = 0.25\times\left(20\times10^6\right) r_i = 10^4$$

$$\ln\frac{r_0}{r_i} = 1, r_0 = e\ r_i = 2.718\times 2\text{mm} = 5.4\text{mm}$$



Boundary conditions (경계 조건)

- Tangential component of E

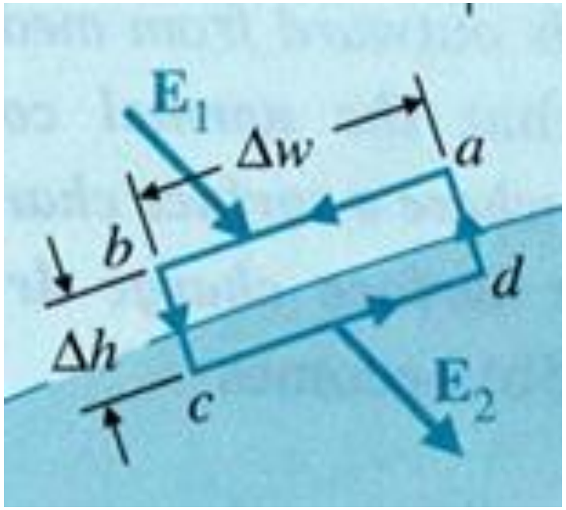
$$\nabla \times \mathbf{E} = 0 \rightarrow E_{1t} = E_{2t}$$

$$\oint_{abcda} \mathbf{E} \cdot dl = 0$$

$$= \mathbf{E}_1 \cdot d\mathbf{w} + \mathbf{E}_2 \cdot -d\mathbf{w}$$

$$= \left(E_{1t} - E_{2t}\right) dw = 0$$

$$E_{1t} = E_{2t}$$



Boundary conditions (경계 조건)

- Normal component of D

$$\nabla \cdot \mathbf{D} = 0 \rightarrow D_{1n} - D_{2n} = \rho_s \ \ (\mathrm{C/m^2})$$

$$\nabla \cdot \mathbf{D} = \rho$$

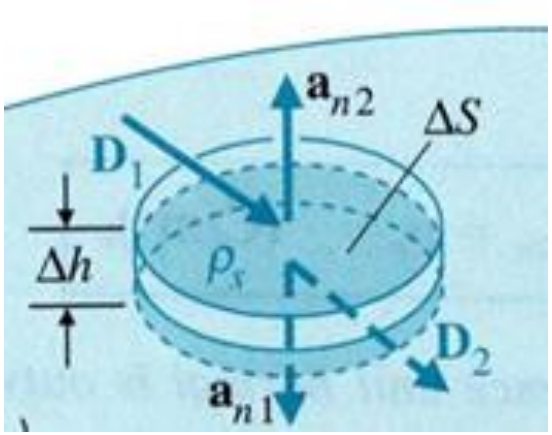
$$\rightarrow \oint_S D \cdot ds = \left(\mathbf{D}_1 \cdot \mathbf{a}_{n2} + \mathbf{D}_2 \cdot \mathbf{a}_{n1}\right)\Delta S$$

$$= \mathbf{a}_{n2} \cdot \left(\mathbf{D}_1 - \mathbf{D}_2\right)\Delta S = \int_V \rho d\upsilon = \rho \Delta S \Delta h$$

$$D_{1n} - D_{2n} = \rho_s$$

$\rho \sim$ constant

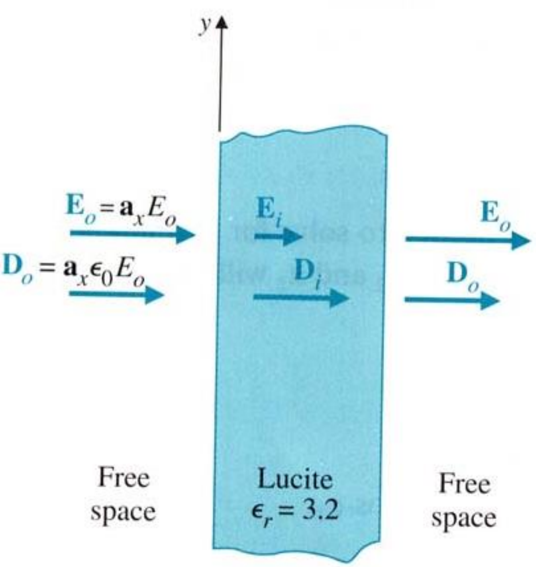
$$\rho_s = \lim_{\Delta h \rightarrow 0} \rho \Delta h$$



- Curl → 수평 성분, divergence → 수직 성분

Boundary conditions

- Example 3.14
  - A lucite sheet (ε_r=3.2) is introduced perpendicularly in a uniform electric field $\mathbf{E_0} = \mathbf{a}_x E_0$ in free space. Determine $\mathbf{E}_i$, $\mathbf{D}_i$, and $\mathbf{P}_i$ inside the lucite.



Boundary condition at the left interface

$$\mathbf{D}_i = \mathbf{a}_x D_i = \mathbf{a}_x D_0 \rightarrow \mathbf{D}_i = \mathbf{a}_x \varepsilon_0 E_0$$

no change in electric flux density across the interface.

$$\mathbf{E}_i = \frac{1}{\varepsilon}\mathbf{D}_i = \frac{1}{\varepsilon_r \varepsilon_0}\mathbf{D}_i = \mathbf{a}_x \frac{E_0}{3.2}$$

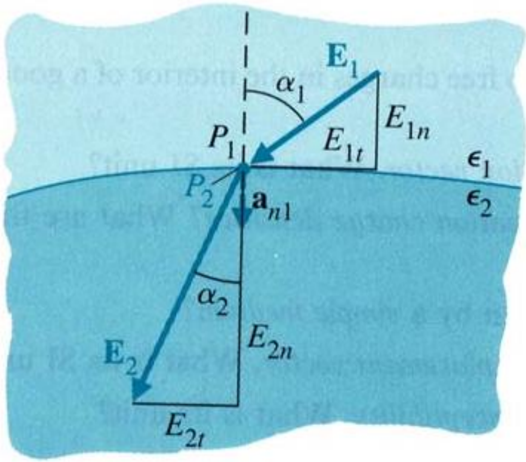
The effect of the lucite sheet is to reduce electric intensity.

$$\mathbf{P}_i = \mathbf{D}_i - \varepsilon_0 \mathbf{E}_i = \mathbf{a}_x\left(1 - \frac{1}{3.2}\right)\varepsilon_0 E_0 = \mathbf{a}_x 0.6875\varepsilon_0 E_0 \quad \left(\mathrm{C/m^2}\right)$$

Does the solution of this problem change if the original electric field is no uniform; that is, if $\mathbf{E}_0 = \mathbf{a}_x E(y)$?

Boundary conditions

- Example 3.15
  - Determine the magnitude and direction of E at point P_2 in medium 2. (Charge-free boundary)



0

$$E_2 \sin\alpha_2 = E_1 \sin\alpha_1 \leftarrow E_{2t} = E_{1t}$$

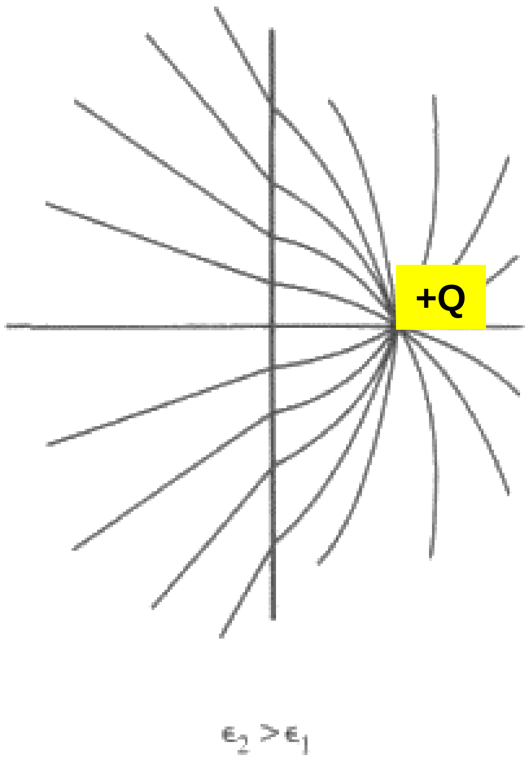
$$\varepsilon_2 E_2 \cos\alpha_2 = \varepsilon_1 E_1 \cos\alpha_1 \leftarrow D_{2n} - D_{1n} = \rho_s$$

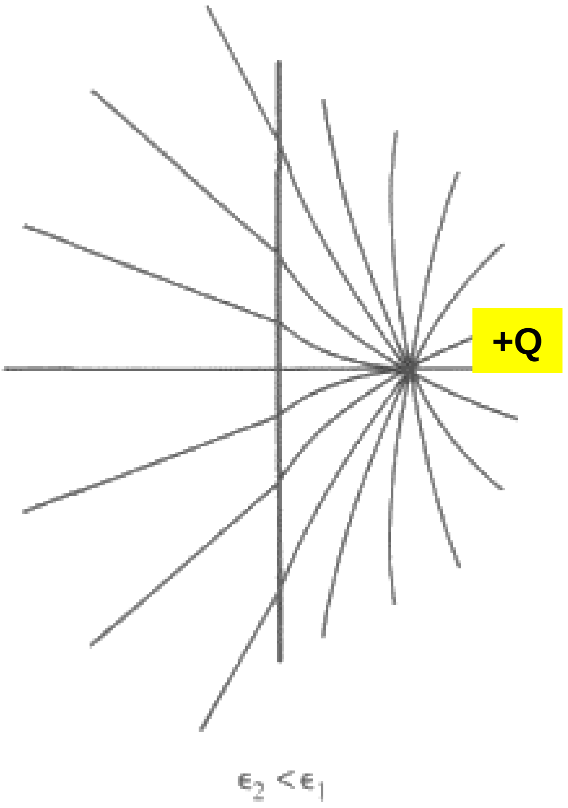
$$E_2 = \sqrt{E_{2t}^2 + E_{2n}^2} = \sqrt{(E_2 \sin\alpha_2)^2 + (E_2 \cos\alpha_2)^2}$$

$$E_2 = \left[(E_1 \sin\alpha_2)^2 + \left(\frac{\varepsilon_1}{\varepsilon_2} E_1 \cos\alpha_1 \right)^2 \right]^{1/2} \text{ or } E_2 = E_1 \left[\sin^2\alpha_1 + \left(\frac{\varepsilon_1}{\varepsilon_2} \cos\alpha_1 \right)^2 \right]^{1/2}$$

Boundary conditions

- E field line



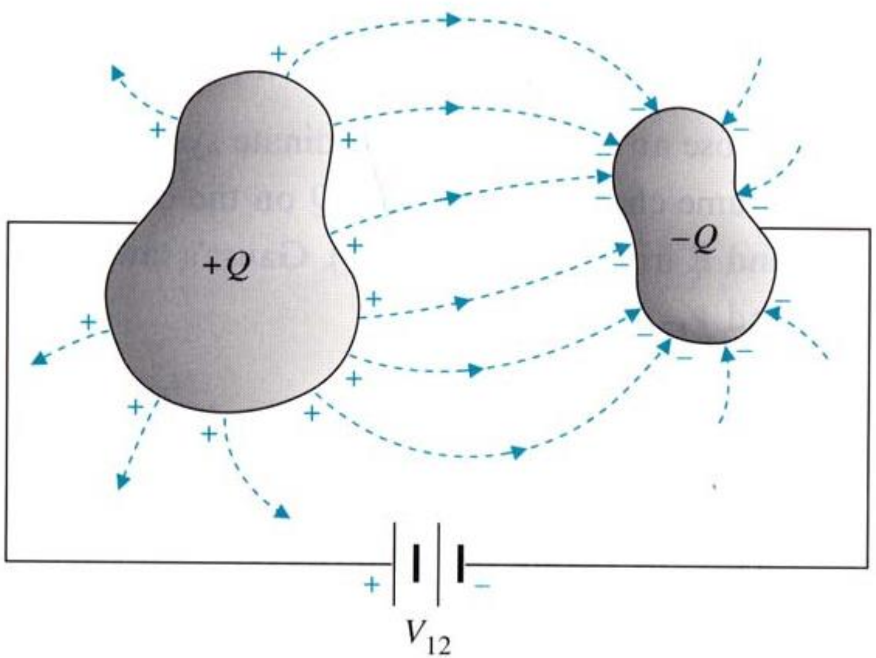


Capacitances and Capacitors

1. Choose an appropriate coordinate system.
2. Assume charges, +*Q*, -*Q*
3. Find E
4. Find V_{12} by evaluating from –*Q* to +*Q*.

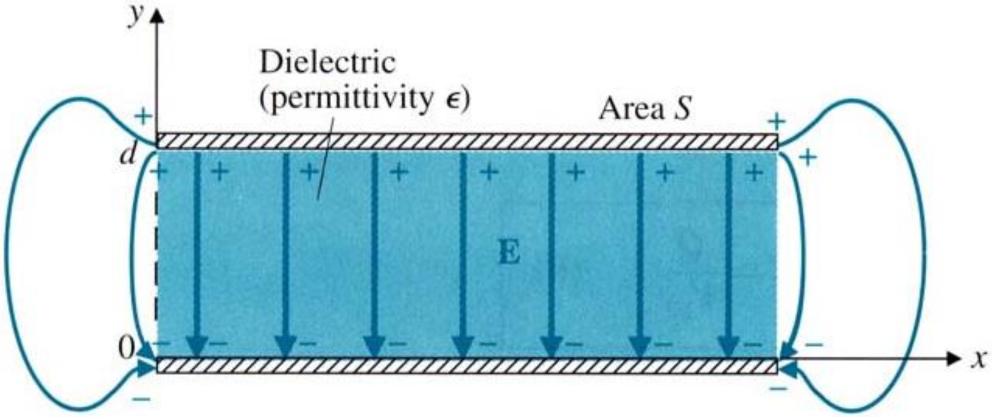
$$V_{12} = -\int_2^1 \mathbf{E} \cdot dl$$

5. Find C by taking the ratio Q/V_{12}



Capacitances and Capacitors

- Example 3-17



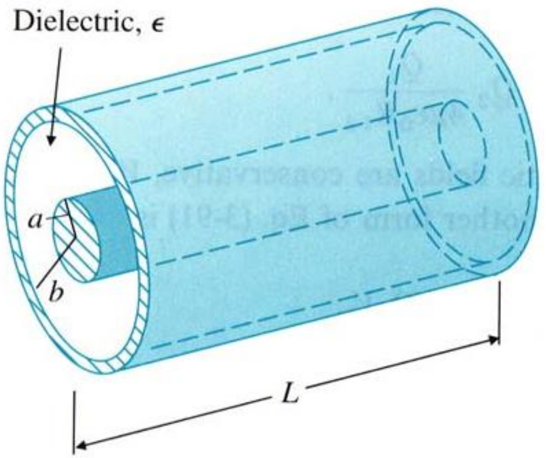
$$\rho_s = \frac{Q}{S} \rightarrow \mathbf{E} = -\mathbf{a}_y \frac{\rho_s}{\varepsilon} = -\mathbf{a}_y \frac{Q}{\varepsilon S}$$

$$V_{12} = -\int_{y=0}^{y=d} \mathbf{E} \bullet d\mathbf{l} = -\int_0^d \left(-\mathbf{a}_y \frac{Q}{\varepsilon S} \right) \bullet \left(\mathbf{a}_y dy \right) = \frac{Q}{\varepsilon S} d$$

$$C = \frac{Q}{V_{12}} = \varepsilon \frac{S}{d}$$

Capacitances and Capacitors

- Example 3-18 : Coaxial cable



Applying Gauss's law to a cylindrical Gaussian surface within the dielectric $a < r < b$

$$\rho_l = \frac{Q}{L}$$

$$\mathbf{E} = \mathbf{a}_r E_r = \mathbf{a}_r \frac{Q}{2\pi\varepsilon r L}$$

$$V_{12} = -\int_{r=b}^{r=a} \mathbf{E} \bullet d\mathbf{l} = -\int_b^a \left(\mathbf{a}_r \frac{Q}{2\pi\varepsilon r L} \right) \bullet \left(\mathbf{a}_r dr \right) = \frac{Q}{2\pi\varepsilon L} \ln\left(\frac{b}{a} \right)$$

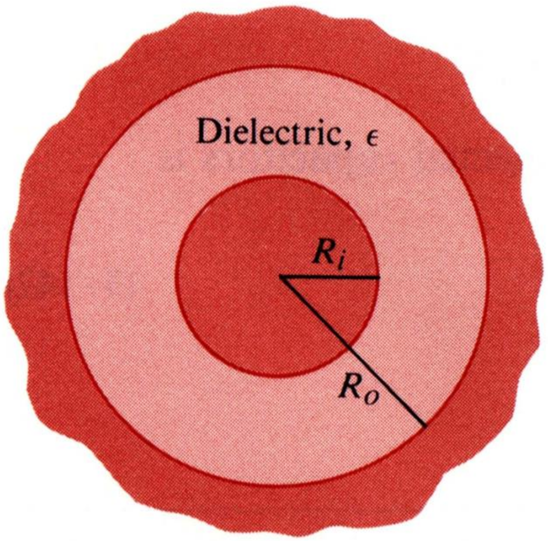
Therefore

$$C = \frac{Q}{V_{ab}} = \frac{2\pi\varepsilon L}{\ln\left(\frac{b}{a} \right)}$$

Earth’s capacitance ? 7.08×10^{-4} (F)

Capacitances and Capacitors

- Spherical capacitor



$$\mathbf{E} = \mathbf{a}_R \frac{Q}{4\pi\varepsilon R^2}$$

$$V = -\int_{R_o}^{R_i} E \bullet dR = \frac{Q}{4\pi\varepsilon}\int_{R_o}^{R_i} \frac{1}{R^2} dR = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{R_i} - \frac{1}{R_o}\right)$$

$$C = \frac{4\pi\varepsilon_0}{1/R_i - 1/R_o}$$

  - One electrode capacitor when $R_o \rightarrow \infty$?

Capacitances and Capacitors

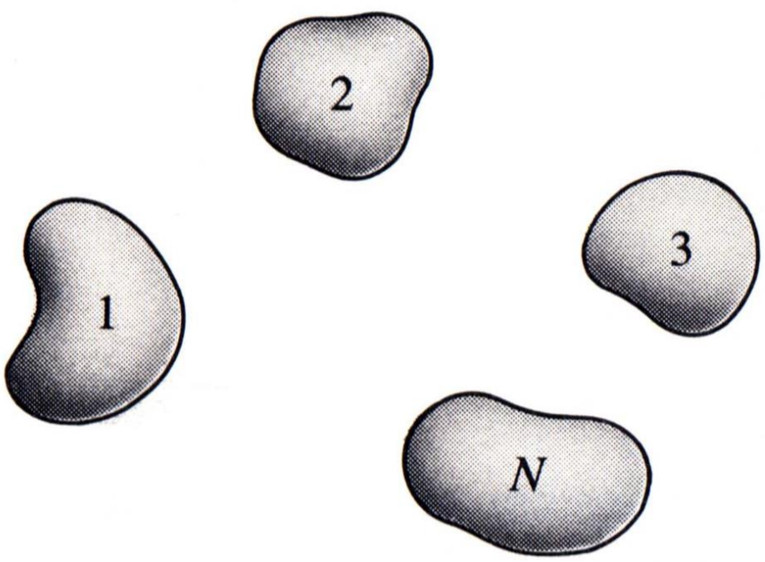
- Multi conductor systems

$$V_1 = p_{11}Q_1 + p_{12}Q_2 + ... + p_{1n}Q_n$$
$$...$$
$$V_n = p_{n1}Q_1 + p_{n2}Q_2 + ... + p_{nn}Q_n$$

$$Q_1 = c_{11}V_1 + c_{12}V_2 + ... + c_{1n}V_n$$
$$...$$
$$Q_n = c_{n1}V_1 + c_{n2}V_2 + ... + c_{nn}V_n$$

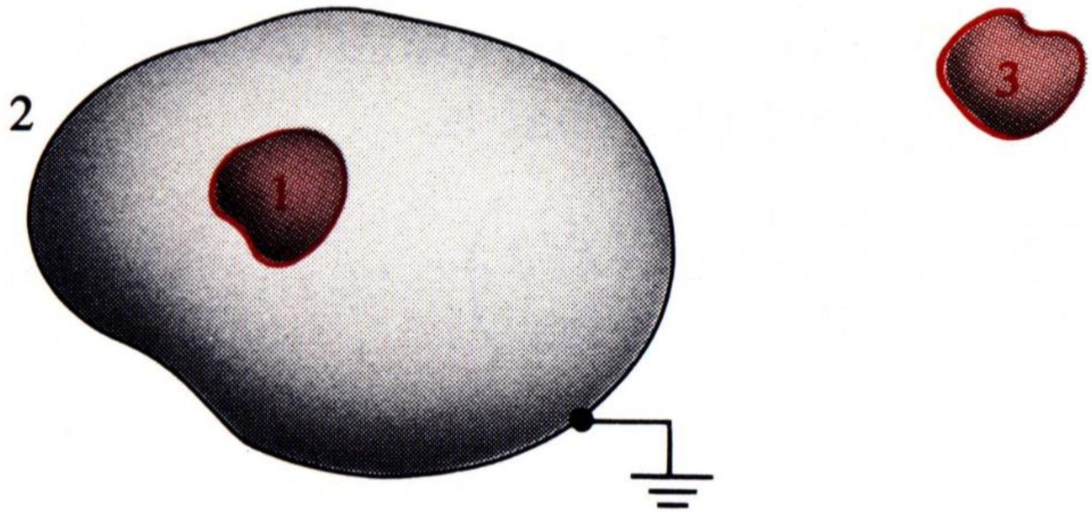
  - For an isolated system

$$Q_1 + Q_2 + ...Q_n = 0$$



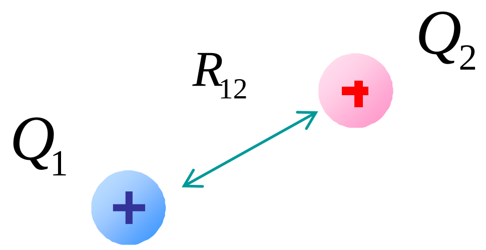
Electrostatic Shielding

- A technique for reducing capacitive coupling between conducting bodies.



Electrostatic Energy

- **Stored Energy**

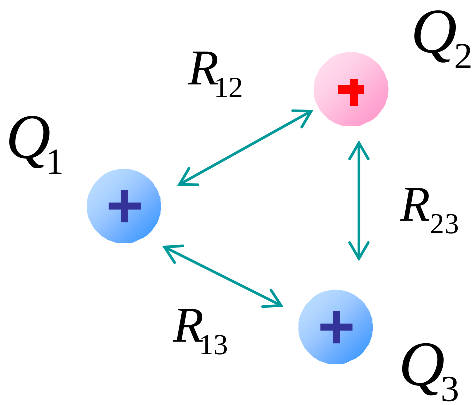


$$W_2 = Q_2 V_2 = Q_2 \frac{Q_1}{4\pi\varepsilon_0 R_{12}} = Q_1 \frac{Q_2}{4\pi\varepsilon_0 R_{12}} = Q_1 V_1$$

$$\therefore W_2 = \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2\right)$$

Electrostatic Energy

- Three charges case



Suppose another charge Q_3 is brought from infinity to a point

$$\Delta W = Q_3 V_3 = Q_3 \left(\frac{Q_1}{4\pi\varepsilon_0 R_{13}} + \frac{Q_2}{4\pi\varepsilon_0 R_{23}} \right)$$

$$W_3 = W_2 + \Delta W = \frac{1}{4\pi\varepsilon_0} \left(\frac{Q_1 Q_2}{R_{12}} + \frac{Q_1 Q_3}{R_{13}} + \frac{Q_2 Q_3}{R_{23}} \right)$$

We can rewrite W_3 in the following form:

$$W_3 = \frac{1}{2}\left[Q_1 \left(\frac{Q_2}{4\pi\varepsilon_0 R_{12}} + \frac{Q_3}{4\pi\varepsilon_0 R_{13}} \right) + Q_2 \left(\frac{Q_1}{4\pi\varepsilon_0 R_{12}} + \frac{Q_3}{4\pi\varepsilon_0 R_{23}} \right) + Q_3 \left(\frac{Q_1}{4\pi\varepsilon_0 R_{13}} + \frac{Q_2}{4\pi\varepsilon_0 R_{23}} \right) \right]$$

$$= \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3 \right)$$

- In general

$$W_n = \frac{1}{2} \sum_{k=1}^{n} Q_k V_k \quad (\mathrm{J})$$

Electrostatic Energy

- For a continuous charge distribution of density.

$$W_n = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k \qquad dQ = \rho_\upsilon d\upsilon$$

$$\rightarrow W_e = \frac{1}{2}\int_{V'} V dQ = \frac{1}{2}\int_{V'} \rho_\upsilon V d\upsilon \quad (\mathrm{J})$$

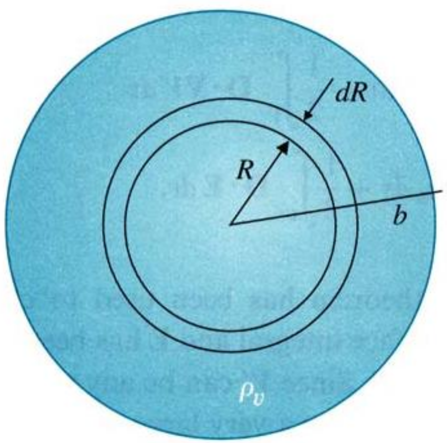
- Example 3.23

$$W_e = \frac{1}{2}\int_{V'} \rho_\upsilon V d\upsilon$$

$$V(R) = -\int_\infty^R \mathbf{E}\bullet d\mathbf{R} = -\int_\infty^b \mathbf{E}\bullet d\mathbf{R} - \int_b^R \mathbf{E}\bullet d\mathbf{R}$$

Electrostatic Energy

- Example 3-22



$W_e = m_e c^2$?

b $\approx 1.7\times10^{-15}$m !!

assuming that the sphere of charge is assembled

by bringing up a sucession of spherical layers of thickness dR

At a radius R, the potential

$$V_R = \frac{Q_R}{4\pi\varepsilon_0 R}$$

where Q_R is the total charge contained in a spehere of radius R:

$$Q_R = \rho_\upsilon \frac{4}{3}\pi R^3$$

The differential change in a spherical layer of thickness dR

$$dQ_R = \rho_\upsilon 4\pi R^2 dR$$

The work or energy in bringing up dQ_R

$$dW_e = V_R dQ_R = \frac{4\pi}{3\varepsilon_0}\rho_\upsilon^2 R^4 dR$$

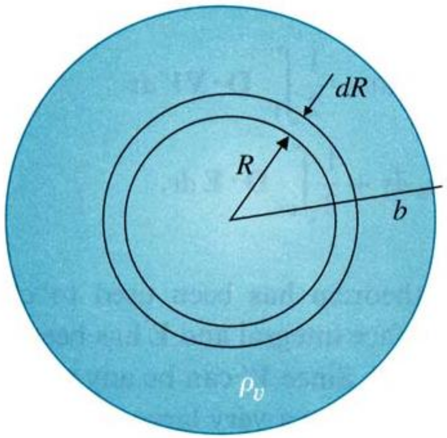
The total work or energy requred to assemble a uniform sphere of charge of radius b and charge density ρ_υ

$$W_e = \int dW_e = \frac{4\pi}{3\varepsilon_0}\rho_\upsilon^2 \int_0^b R^4 dR = \frac{4\pi\rho_\upsilon^2 b^5}{15\varepsilon_0} \rightarrow Q = \rho_\upsilon \frac{4\pi}{3} b^3$$

$$W_e = \frac{3Q^2}{20\pi\varepsilon_0 b} \quad \text{(J)}$$

Electrostatic Energy

- Example 3-23



$$W_e = \frac{1}{2}\int_{V'} \rho_\upsilon V d\upsilon$$

$$V(R) = -\int_{\infty}^{R} \mathbf{E} \bullet d\mathbf{R} = -\int_{\infty}^{b} \mathbf{E} \bullet d\mathbf{R} - \int_{b}^{R} \mathbf{E} \bullet d\mathbf{R}$$

Electrostatic Energy

- In terms of field quantities

$$W_e = \frac{1}{2}\int_{V'} \nabla \cdot \mathbf{D} V d\upsilon \leftarrow \nabla \cdot (V\mathbf{D}) = V\nabla \cdot \mathbf{D} + \mathbf{D} \cdot \nabla V$$

$$W_e = \frac{1}{2}\int_{V'} \nabla \cdot (V\mathbf{D})\, d\upsilon - \frac{1}{2}\int_{V'} \mathbf{D} \cdot \nabla V d\upsilon$$

$$= \frac{1}{2}\oint_{S'} V\mathbf{D} \cdot \mathbf{a}_n ds + \frac{1}{2}\int_{V'} \mathbf{D} \cdot \mathbf{E} d\upsilon$$

$$V \propto \frac{1}{R}, D \propto \frac{1}{R^2}$$

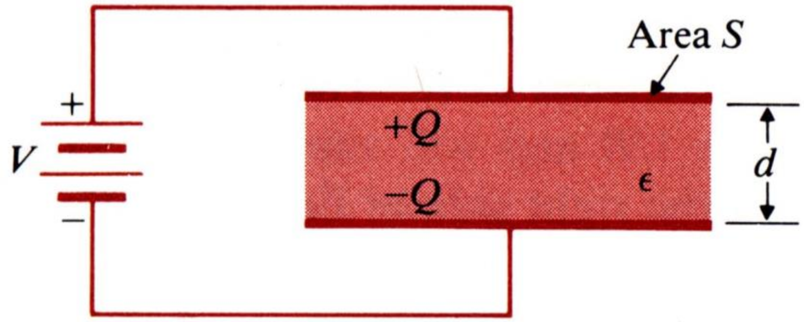
$$W_e = \frac{1}{2}\int_{V'} \mathbf{D} \cdot \mathbf{E} d\upsilon \leftarrow \mathbf{D} = \varepsilon \mathbf{E}$$

$$W_e = \frac{1}{2}\int_{V'} \varepsilon E^2 d\upsilon$$

Electrostatic Energy

- Example 3.24

$$W_e = \frac{1}{2} C V^2$$



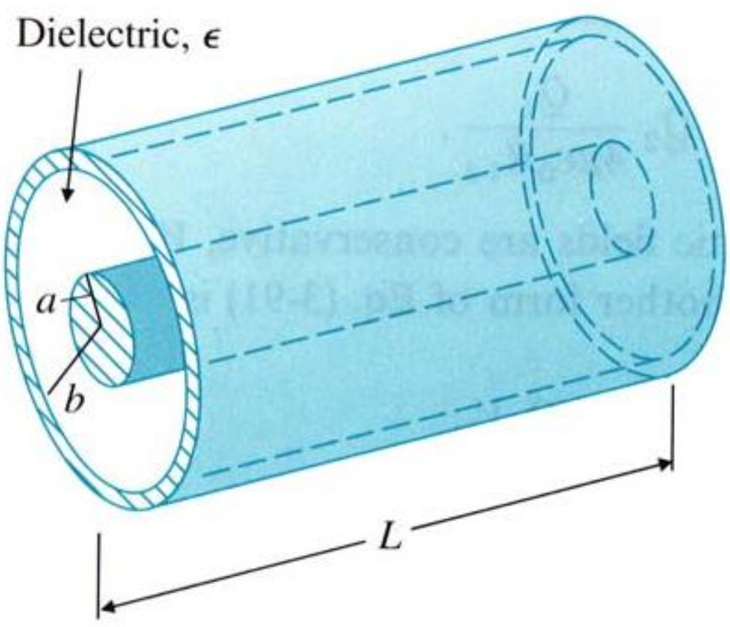
$$W_e = \frac{1}{2}\int_0^d \varepsilon \left(\frac{V}{d}\right)^2 dv = \frac{1}{2}\varepsilon\left(\frac{V}{d}\right)^2 Sd = \frac{1}{2}\left(\varepsilon\frac{S}{d}\right)V^2$$

Electrostatic Energy

- Example 3.25 (coaxial cable)

$$W_e = \frac{1}{2}\int_a^b \varepsilon \left(\frac{Q}{2\pi\varepsilon L r} \right)^2 L 2\pi r dr = \left(\frac{Q^2}{4\pi\varepsilon L} \right) \int_a^b \frac{1}{r} dr = \frac{Q^2}{4\pi\varepsilon L} \ln\frac{b}{a}$$

$$= \frac{Q^2}{2C}$$



$$C = \frac{2\pi\varepsilon L}{\ln\left(\frac{b}{a}\right)}$$

Electrostatic Forces

- Principle of virtual displacement.
  - Force

Mechanical work done by the system (constant Q)

$$dW = F_Q \cdot dl$$

$$dW = -dW_e$$

$$dW_e = \left(\nabla W_e\right) \cdot dl \rightarrow F_Q = -\nabla W_e \quad (\mathrm{N})$$

$$\left(F_Q\right)_x = -\frac{\partial W_e}{\partial x} \quad \leftrightarrow \quad \left(F_V\right)_x = \frac{\partial W_e}{\partial x}$$

constant V

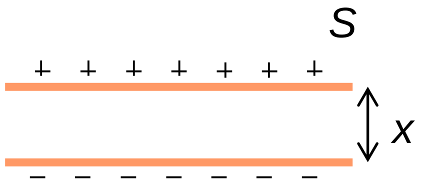
  - Torque

$$\left(T_z\right)_Q = -\frac{\partial W_e}{\partial \phi} \leftrightarrow \left(\mathrm{T_z}\right)_V = \frac{\partial W_e}{\partial \phi} \quad \leftarrow dW = T_{Qz} d\phi$$

constant V

Electrostatic Forces

- Example 3.26



$$W_e = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

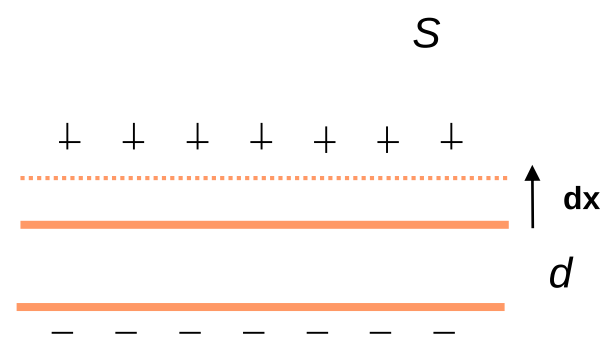
$$\mathbf{E} = -\mathbf{a}_x \frac{\rho_s}{\varepsilon_0} = -\mathbf{a}_x \frac{Q}{\varepsilon_0 S} \rightarrow V = -\int_{\text{lower plate}}^{\text{upper plate}} \mathbf{E} \bullet \mathbf{a}_x dx = \frac{Q}{\varepsilon_0 S} x$$

$$\left(F_Q\right)_x = -\frac{Q}{2}\frac{\partial V}{\partial x} = -\frac{Q^2}{2\varepsilon_0 S} = -Q\frac{Q}{2\varepsilon_0 S} = -Q\frac{\rho}{2\varepsilon_0} = -QE$$

– Fixed potentials ?

Electrostatic Forces

- Constant Q
  - Constant E
  - Increase in W_E $$W_E = \frac{Q^2}{2C}$$
  - $dW + dW_E = 0$
  - Additional energy from ?



- Constant V
  - Decrease in E $$W_E = \frac{1}{2}CV^2$$
  - $E \propto 1/d$, decrease in W_E
  - Work done by battery, $dW_B = VdQ$
  - $dW_B = 2\, dW_E$ (= ½ dQ V)
  - $dW + dW_E = dW_B = dW_E \rightarrow dW = dW_E$

Summary

- Electrostatics

Charge density, ρ

$$V = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho}{R}dv$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho\hat{\mathbf{R}}}{R^2}dv$$

$$\nabla\bullet\mathbf{E} = \rho/\varepsilon_0,$$

$$\nabla\times\mathbf{E} = 0$$

$$\nabla^2 V = -\rho/\varepsilon_0$$

Potential, V

$$\mathbf{E} = -\nabla V$$

$$V = -\int \mathbf{E}\bullet d\mathbf{l}$$

Electric Field, E

Home Work

- 3장
  - 5,6,8,11,12,15,16,19,20,22,23,27,28,32,33,35,41,44,48

Discussion Questions

- Electric lines of force never cross. Why?
- The free electrons in a metal are gravitationally attracted toward the earth. Why, then, don't they all settle to the bottom of the conductor, like sediment settling to the bottom of a river?
- Some of the free electrons in a good conductor (such as a piece of copper) move at speeds of $10^6 \mathrm{m}/s$ or move. Why do these electrons not fly out of the conductor completely?
- You have a negatively charged object. How can you use it to place a net negative charge on an insulated metal sphere? To place a net positive charge on the sphere?
- The electric force between an electron and a proton, between two electrons, or between two protons is much stronger than the gravitational force between any of these pairs of particles. Yet even though the sun and planets contain electrons and protons, it is the gravitational force that holds the planets in their orbits around the sun. Explain this seeming contradiction.
- A spherical Gaussian surface encloses a point charge q. If the point charge is moved from the center of the sphere to a point away from the center, does the electric field at a point on the surface change? Dose the total flux through the Gaussian surface change? Explain.
- Are Coulomb's law and Gauss's law *completely* equivalent? Are there any situations in electrostatics in which one is valid and the other is not? Explain your reasoning.
- If the electric field of a point charge were proportional to $1/r^3$ instead of $1/r^2$, would Gauss's law still be valid? Explain your reasoning. (*Hint*: Consider a spherical Gaussian surface centered on a single point charge.)
- The electric field $\vec{E}$ is uniform throughout a certain region of space. A small conducting sphere that carries a net charge Q is then placed in this region. What is the electric field inside the sphere? Explain your reasoning.

Discussion Questions

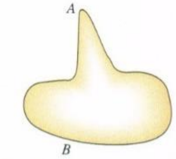


Figure 22.31 Question Q22.12.

- The magnitude of $\vec{E}$ at the surface of an irregularly shaped solid conductor must be greatest in regions where the surface curves most sharply, such as point A in Fig. 22.31, and must be least in flat regions such as point B in Fig. 22.31. Explain why this must be so by considering how electric field lines must be arranged near a conducting surface. How does the surface charge density compare at points A and B? Explain.
- A solid conductor has a cavity in its interior. Would the presence of a point charge inside the cavity affect the electric field outside the conductor? Why or why not? Would the presence of a point charge outside the conductor affect the electric field inside the cavity? Again, why or why not?
- You find a sealed box on your doorstep. You suspect that the box contains several charged metal sphere packed in insulating material. How can you determine the total net charge inside the box without opening the box? Or is this not possible?
- A solid copper sphere has a net positive charge. The charge is distributed uniformly over the surface of the sphere and the electric field inside the sphere is zero. Then a negative point charge outside the sphere is brought close to the surface of the sphere. Is all the net charge on the sphere still on its surface? If so, is this charge still distributed uniformly over the surface? If it is not uniform, how is it distributed? Is the electric field inside the sphere still zero? In each case justify your answers. Suppose that a Gaussian surface encloses no net charge. Does Gauss' law require that $\mathbf{E}$ equal zero for all points on the surface? Is the converse of this statement true; that is, if $\mathbf{E}$ equals zero everywhere on the surface, does Gauss's law require that there be no net charge inside?
- Is $\mathbf{E}$ necessarily zero inside a charged rubber balloon if the balloon is (a) spherical or (b) sausage shaped? For each shape, assume the charge to be distributed uniformly over the surface. How would the situation change, if at all, if the balloon has a thin layer of conducting paint on its outside surface?
- A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blown up, how does E vary from points (a) inside the balloon, (b) at the surface of the balloon, and (c) outside the balloon?

Discussion Questions

- In section 25-9, the *total* charge on the infinite rod is infinite. Why is not *E* also infinite? After all, according to Coulomb's law, if q is infinite, so is *E*.
- The field due to an infinite sheet of charge is uniform, having the same strength at all points no matter how far from the surface charge. Explain how this can be, given the inverse square nature of Coulomb's law.
- As you penetrate a uniform sphere of charge, *E* should decrease because less charge lies inside a sphere drawn through the observation point. On the other hand, *E* should increase because you are closer to the center of this charge. Which effect dominates and why?
- Is it possible to have an arrangement of two point charges separated by a finite distance such that the electric potential energy of the arrangement is the same as if the two charges were infinitely far apart? Why or why not? What if there are three charges? Explain your reasoning.
- If $\vec{E}$ is zero everywhere along a certain path that leads from point A to point B, what is the potential difference between those two points? Does this mean that $\vec{E}$ is zero everywhere along any path from A to B? Explain.
- If $\vec{E}$ is zero throughout a certain region of space, is the potential necessarily also zero in this region? Why or why not? If not, what can be said about the potential?
- If you carry out the integral of the electric field $\int \vec{E} \cdot d\vec{l}$ for a closed path like that shown in Fig. 23.27, the integral will always be equal to zero, independent of the shape of the path and independent of where charges may be located relative to the path. Explain why.

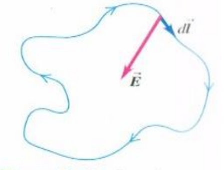


Figure 23.27 Question Q23.7.

- It is easy to produce a potential difference of several thousand volts between your body and the floor by scuffing your shoes across a nylon carpet. When you touch a metal doorknob, you get a mild shock. Yet contact with a power line of comparable voltage would probably be fetal. Why is there a difference?

Discussion Questions

- If the electric potential at a single point is known, can $\vec{E}$ at that point be determined? If so, how? If not, why not?
- A conducting sphere is to be charged by bringing in positive charge a little at a time until the total charge is Q. The total work required for this process is alleged to be proportional to Q^2. Is this correct? Why or why not?
- A conductor that carries a net charge Q has a hollow, empty cavity in its interior. Does the potential vary from point to point within the material of the conductor? What about within the cavity? How does the potential inside the cavity compare to the potential within the material of the conductor?
- A positive point charge is placed near a very large conducting plane. A professor of physics asserted that the field caused by this configuration is the same as would be obtained by removing the plane and placing a negative point charge of equal magnitude in the mirror-image position behind the initial position of the plane. Is this correct? Why or why not? (*Hint*: Inspect Fig. 23.23b.)
- In electronics it is customary to define the potential of ground (thinking of the earth as a large conductor) as zero. Is this consistent with the fact that the earth has a net electric charge that is not zero? (Refer to Exercise 21.30.)
- Suppose that the earth has a net charge that is not zero. Why is it still possible to adopt the earth as a standard reference point of potential and to assign the potential V=0 to it?
- If you know **E** only at a given point, can you calculate **V** at that point? If not, what further information do you need?
- If the surface of a charged conductor is an equipotential, does that mean that charge is distributed uniformly over that surface? If the electric field is constant in magnitude over the surface of a charged conductor, does that mean the charge is distributed uniformly?
- An isolated conducting spherical shell carries a negative charge. What will happen if a positively charged metal object is placed in contact with the shell interior? Discuss the three cases in which the positive charge is (a) less than, (b) equal to, and (c) greater than the negative charge in magnitude.

Discussion Questions

- A capacitor is connected across a battery. (a) Why does each plate receive a charge of exactly the same magnitude? (b) Is this true even if the plates are of different sizes?
- A sheet of aluminum foil of negligible thickness is placed between the plates of a capacitor as in Fig.17. What effect has it on the capacitance if (a) the foil is electrically insulated and (b) the foil is connected to the upper plate?
- If you were not to neglect the fringing of the electric field lines in a parallel-plate capacitor, would you calculate a higher or a lower capacitance?
- A parallel-plate capacitor is charged by using a battery, which is then disconnected. A dielectric slab is then slipped between the plates. Describe qualitatively what happens to the charge, the capacitance, the potential difference, the electric field, and the stored energy.
- While a parallel-plate capacitor remains connected to a battery, a dielectric slab is slipped between the plates. Describe qualitatively what happens to the charge, the capacitance, the potential difference, the electric field, and stored energy. Is work required to insert the slab?
- Two identical capacitors are connected as shown in Fig 19. A dielectric slab is slipped between the plates of one capacitor, the battery remaining connected. Describe qualitatively what happens to the charge, the capacitance, the potential difference, the electric field, and the stored energy for each capacitor.
- Equation (24.2) shows that the capacitance of a parallel-plate capacitor becomes larger as the plate separation *d* decreases. However, there is a practical limit to how small *d* can be made, which places limits on how large *C* can be. Explain what sets the limit on d.(Hint: What happens to the magnitude of the electric field as d->0?)
- Suppose the two plates of a capacitor is charged by connecting it to a battery, do the charges on the two plates have equal magnitude, or may they be different? Explain your reasoning.
- A parallel-plate capacitor is charged by being connected to a battery and is kept connected to the battery. The separation between the plates is then doubled. How does the electric field change? The charge on the plates? The total energy? Explain your reasoning.

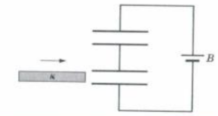


Figure 19 Question 20.

Discussion Questions

- According to the text, we can consider the energy in a charged capacitor to be located in the field between the plates. But suppose there is vacuum between the plates; can there be energy in a vacuum? Why or why not? What form could this energy take ?
- The charged plates if a capacitor attract each other, so to pull the plates farther apart requires work by some external force. What becomes of the energy added by this work? Explain your reasoning.
- The two plates of a capacitor are given charges $\pm Q$. The capacitor is then disconnected from the charging device so that the charges on the plates can't change, and the capacitor is immersed in a tank of oil. Does the electric field between the plates increase, decrease, or stay the same? Explain your reasoning. How can this field measured?
- A conductor is an extreme case of a dielectric, since if an electric field is applied to a conductor, charges are free to move within the conductor to set up "induced charges." What is the dielectric constant of a perfect conductor? Is it K=0? K=∞? Or something in between? Explain your reasoning.
- A capacitor of capacitance C is charged to a potential difference V_0. The terminals of the charged capacitor are then connected to those of an uncharged capacitance C. Compute a) the original of the system; b) the final potential difference across each capacitor; c) the final energy of the system; d) the decrease in energy when the capacitors are connected. e) Where did the "lost" energy go?